particle physics and cosmology from almost-commutative manifolds

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outline

- motivation
- formalism: noncommutative spectral geometry
- physical meaning of the doubling of the algebra
- particle physics phenomenology: geometric explanation of SM
- gravitational theory $\Longrightarrow$ early universe cosmology
- conclusions

motivation

cosmology

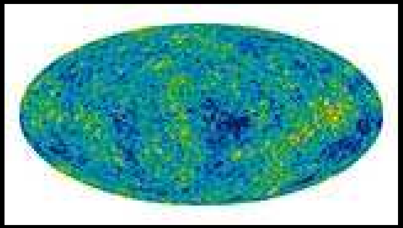
particle physics

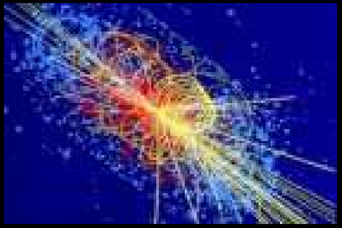
cosmology

cosmology

EU models tested with

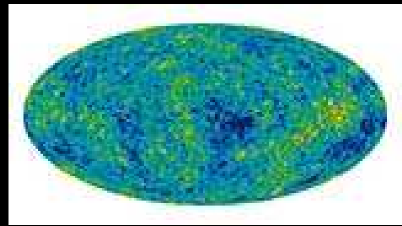
- astrophysical data (CMB)
- high energy experiments (LHC)





cosmology

EU models tested with



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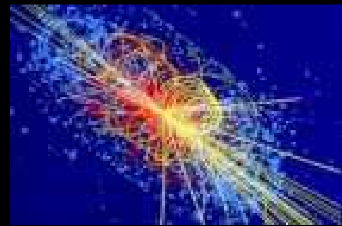
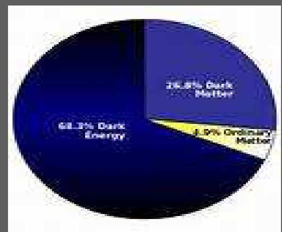
- high energy experiments (LHC)

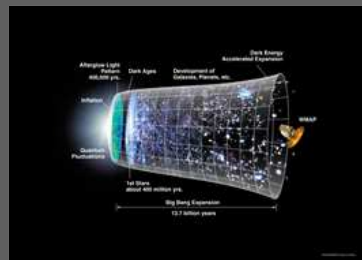


despite the golden era of cosmology, a number of questions:

- origin of DE / DM



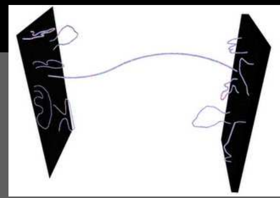
- search for natural and well-motivated inflationary model (alternatives...)



. . .

are still awaiting for a definite answer main approaches:

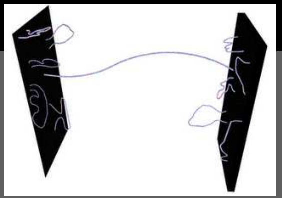
- string theory



- LQC, SF, WdW, CDT, CS,...

main approaches:

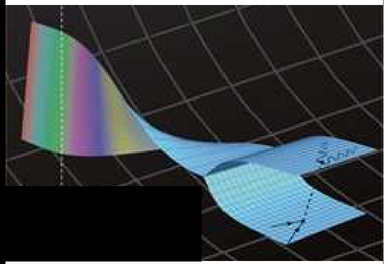
- string theory



- LQC, SF, WdW, CDT, CS,...

- noncommutative spectral geometry

$$\mathcal{S}^{\mathrm{E}} = \int \Big(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^\star R^\star + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^\alpha_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \Big) \sqrt{g}\, d^4x \ ,$$



particle physics

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at low energies:

$$\boxed{S_{\text{Einstein--Hilbert}} + S_{\text{Standard Model}}}$$

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GR
diffeomorphism invariance

gauge symmetries
local gauge invariance

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this difference may be responsible for difficulty in finding a unified theory of all interactions including gravity in addition:

- why G is $U(1) \times SU(2) \times SU(3)$?
- why fermions occupy the representations they do ?
- why 3 families / why 16 fundamental fermions per each ?
- what is the origin of Higgs mechanism and SSB ?
- what is the Higgs mass and how are explained all fermionic masses?

…

to be answered by the ultimate unified theory of all interactions noncommutative spectral geometry

goal

answer one of the most outstanding quests in theoretical physics: the unification of the four fundamental forces

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price to pay: unification obtained only at classical level (same for any gauge theory + quantisation of gravity) goal

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noncommutative space times naturally appear in string theory and QG (in LQG the quantised area operator is a manifestation of an underlying quantum geometry)

NCG: curved spacetime with an uncertainty principle

aim:

obtain all forces as pseudo-forces from some general "coordinate transformations" acting on some general "spacetime"

- NCG : bottom-up approach

guess small-scale structure of ST from knowledge at EW scale

- string theory : top-down approach

derive SM directly from planck scale physics

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QG $\Longrightarrow$ ST is wildly noncommutative manifold at very high E

at an intermediate scale the algebra of coordinates is only a mildly noncommutative algebra of matrix valued functions

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QG → ST is wildly noncommutative manifold at very high E

at an intermediate scale the algebra of coordinates is only a mildly noncommutative algebra of matrix valued functions

suitably chosen → SM coupled to gravity

- string theory : top-down approach

derive SM directly from planck scale physics

geometrisation of physics

- kaluza-klein theories: electromagnetism described by circle bundle over spacetime manifold, connection = em potential

- yang-mills gauge theories: bundle geometry over spacetime, connections = gauge potentials

- string theory: 6 extra dimensions (calabi-yau) over spacetime, string vibrations = types of particles

- NCG models: extra dimensions are NC spaces, pure gravity on product space becomes gravity + matter on spacetime

the language of nomcommutative geometry

- space $\longrightarrow$ functions
- topological space $\longrightarrow$ continuous functions
- measure $\longrightarrow$ measurable functions
- differentiation $\longrightarrow$ smooth functions
- vector fields $\longrightarrow$ differential forms
- differential operators $\longrightarrow$ operators on a hilbert space

comment

"nothing" to do with $[\mathbf{x}^i, \mathbf{x}^j] = i\theta^{ij}$ used to implement fuzziness of space-time

anti-symmmetric real dxd matrix

however

euclidean version of moyal NCFT → spectral triples formulation of NCG

noncompact noncommutative spin manifold

compact noncommutative spin manifold

gayral, gracia-bondia, iochum, schücker, varilly (2004)

noncommutative spectral geometry

SM as a phenomenological model, which dictates geometry of ST, so that the maxwell-dirac action produces the SM

noncommutative spectral geometry

SM as a phenomenological model, which dictates geometry of ST, so that the maxwell-dirac action produces the SM

geometric space defined by the product $\mathcal{M} \times \mathcal{F}$ of a continuum compact riemannian manifold $\mathcal{M}$ and a tiny discrete finite noncommutative space $\mathcal{F}$

geometry: tensor product of an internal (zero-dim) geometry for the SM and a continuous geometry for space-time

noncommutative spectral geometry

SM as a phenomenological model, which dictates geometry of ST, so that the maxwell-dirac action produces the SM

geometric space defined by the product $\mathcal{M} \times \mathcal{F}$

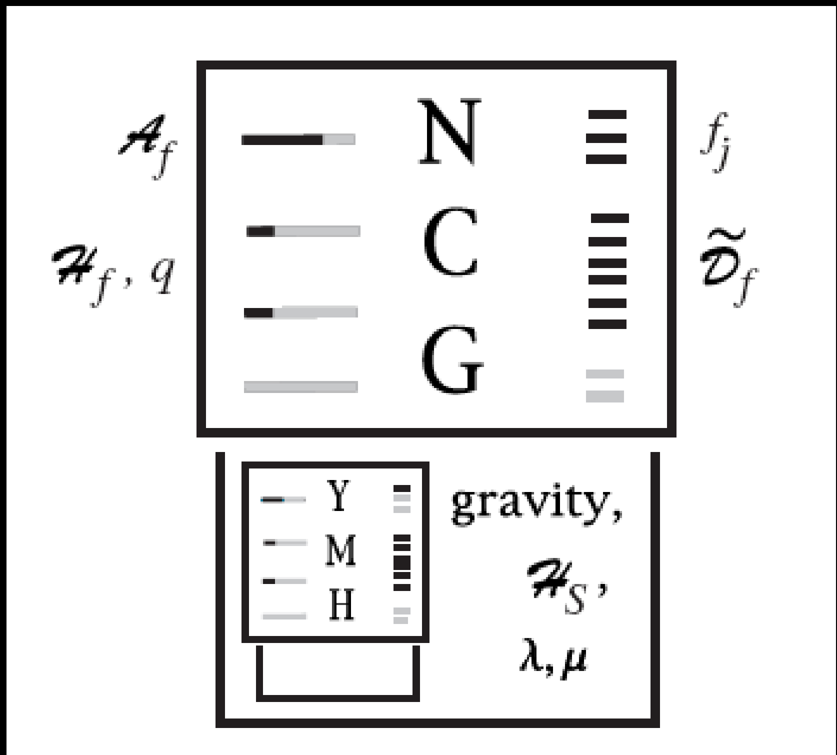
almost commutative geometry

4-dim ST with an "internal" kaluza-klein space attached to each point

the "fifth" dim is a discrete, 0-dim space central idea: characterise ordinary riemaniann manifolds by spectral data central idea: characterise ordinary riemaniann manifolds by spectral data

input

output



lagrange density →
via perturbative QFT compute
particle phenomenology

compare with experiments to see
if your input wins or loses

alain connes' slot machine

spin manifolds in NCG

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$\mathcal{M}$: compact 4dim riemanian spin manifold

$C^\infty(\mathcal{M})$: set of smooth infinitely (differentiable) functions;

.

$\mathcal{H} = L^2(\mathcal{M}, S)$: Hilbert space of square-integrable spinors S on $\mathcal{M}$

spin manifolds in NCG

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$C^\infty(\mathcal{M})$: set of smooth infinitely (differentiable) functions; they form an algebra under pointwise multiplication

$\mathcal{H} = L^2(\mathcal{M}, S)$: Hilbert space of square-integrable spinors S on $\mathcal{M}$

the algebra $\mathcal{A} = C^\infty(\mathcal{M})$ acts on $\mathcal{H}$ as multiplication operators by pointwise scalar multiplication

$$(f, \psi)(x) := f(x)\psi(x)$$

for a function f and a spinor field ψ

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$\not{D}$: the Dirac operator $-i\gamma^\mu \nabla^S_\mu$ in terms of Dirac gamma matrices γ^μ and the spin (Levi-Civita) connection ∇^S ; it acts as first order differential operator on the spinors ψ

spin manifolds in NCG

$$(C^\infty(M), L^2(M,S), \not{D})$$

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almost-commutative manifolds

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the canonical triple encodes the structure of the spacetime $\mathcal{M}$,
the finite space $\mathcal{F}$ will encode the internal degrees of freedom
at each point in spacetime

$$F := (\mathcal{A}_F, \mathcal{H}_F, D_F)$$

the internal degrees of freedom will lead to the description of
a gauge theory on $\mathcal{M}$

almost-commutative manifolds

product of a spin manifold by a finite space, which in general may be noncommutative $\mathcal{M} \times \mathcal{F}$

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almost-commutative (spin) manifold:

$$M \times F := (C^\infty(M, \mathcal{A}_F), L^2(M, S) \otimes \mathcal{H}_F, \not{D} \otimes \mathbb{I} + \gamma_5 \otimes D_F)$$

$$D := \not{D} \otimes \mathbb{I} + \gamma_5 \otimes D_F$$

Dirac operator of the almost-commutative manifold spectral geometry given by the product rules: $\mathcal{M} \times \mathcal{F}$

$$\mathcal{A} = \underbrace{C^\infty(\mathcal{M}, C)}_{} \otimes \mathcal{A}_\mathcal{F}$$

algebra of smooth complex valued functions on euclidean 4dim $\mathcal{M}$

$$D = \underbrace{D_\mathcal{M}}_{} \otimes 1 + \gamma_5 \otimes \underbrace{D_\mathcal{F}}_{}$$

Dirac operator on Riemannian spin manifold $\mathcal{M}$

self-adjoint fermion mass matrix

$$\partial\!\!\!/_\mathcal{M} = \sqrt{-1}\gamma^\mu \nabla^s_\mu$$

$$\mathcal{H} = \underbrace{L^2(\mathcal{M}, S)}_{} \otimes \underbrace{\mathcal{H}_\mathcal{F}}_{}$$

space of square integrable Dirac spinors over $\mathcal{M}$

finite dim space, which describes physical particle d.o.f. (helicity, chirality, flavour, charge, ...)

data in the triple describing an almost-commutative manifold

Data	Spin manifold M	Finite space F
Algebra $\mathcal{A}$	Coordinate functions	Internal structure
Hilbert space $\mathcal{H}$	Spinor fields	Particle content
Operator D	Dirac operator $\not{D}$	Yukawa mass matrix D_F

the action of the finite algebra on the fermions will determine their gauge interactions

NCSG based on 3 ansatz:

1. at some energy level, ST is the product $\mathcal{M} \times \mathcal{F}$

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real spectral triple

$$\mathcal{F} = \quad (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}})$$

involutive algebra

self-adjoint operator

(information carried by metric)

complex Hilbert space carrying a representation of the algebra

focus on $D_{\mathcal{F}}$ instead of $g_{\mu\nu}$

11. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

left-right symmetric algebra $\mathbb{C} \oplus \mathbb{H}_{\mathrm{L}} \oplus \mathbb{H}_{\mathrm{R}} \oplus M_3(\mathbb{C})$

$\mathbb{H} \subset M_2(\mathbb{C})$

$$\mathbb{H} = \left\{ \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} \; ; \; \alpha, \beta \in \mathbb{C} \right\}$$

II. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

left-right symmetric algebra

$$\mathbb{C} \oplus \mathbb{H}_{\mathrm{L}} \oplus \mathbb{H}_{\mathrm{R}} \oplus M_3(\mathbb{C})$$

$$\mathbb{H} \subset M_2(\mathbb{C}) \qquad \mathbb{H} = \left\{ \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix} ; \alpha, \beta \in \mathbb{C} \right\}$$

however

- to account for massive neutrinos & neutrino oscillations

⇒ cannot be left-right symmetric model

- NCG imposes constraints on algebras of operators in Hilbert space
- avoid fermion doubling

⇓

11. the finite dimensional algebra $\mathcal{A}_{\mathcal{F}}$ is (main input):

$$\boxed{\mathcal{A}_{\mathcal{F}} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})}$$

$k = 2a$

algebra of quaternions

algebra of complex $k \times k$ matrices

chamseddine and connes (2007)

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algebra of complex $k \times k$ matrices

$k = 2a$

$k = 4$ first value that produces correct # of fermions per generation

prediction

- the number of fermions is the square of an even integer
- the existence of 3 generations is a physical input

chamseddine and connes (2007)

III. spectral action principal:

idea behind spectral action:

while the topology is encoded by the algebra, all other information (e.g., metric) is encoded by the generalised covariant Dirac operator

$$\mathcal{D}_A = \mathcal{D}_{\mathcal{F}} + A + \epsilon' J A J^{-1}$$

where $A = A^{\star}$ self-adjoint operator $A = \sum_j a_j [\mathcal{D}_{\mathcal{F}}, b_j]$, $a_j, b_j \in \mathcal{A}_{\mathcal{F}}$

characterised completely by its spectrum

$$J^2 = \varepsilon \quad , \quad J\gamma = \varepsilon'' \gamma J$$

$$\varepsilon, \varepsilon', \varepsilon'' \in \{\pm 1\}$$

III. spectral action principal:

the action functional depends only on the spectrum of the Dirac operator and is of the form:

$$Tr(f(D_A/\Lambda))$$

cut-off function

fixes the energy scale

positive function that falls to zero at large values of its argument $\Rightarrow$

$\int_0^\infty f(u)udu \ , \quad \int_0^\infty f(u)du$

finite

chamseddine and connes (1996, 1997)

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bosonic part

chamseddine and connes (1996, 1997)

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the action functional depends only on the spectrum of the Dirac operator and is of the form:

$$Tr(f(D_A/\Lambda)) + \frac{1}{2}\langle J\Psi, \mathcal{D}\ \Psi\rangle \ , \quad \Psi \in \mathcal{H}_{\mathcal{F}}^{+}$$

chamseddine and connes (1996, 1997)

III. spectral action principal:

the action functional depends only on the spectrum of the Dirac operator and is of the form:

$$Tr(f(D_A/\Lambda))$$

bosonic part

action sums up eigenvalues of $\mathcal{D}_A$ which are smaller than Λ

evaluate trace with heat kernel techniques (seeley-de witt coefficients)

$$\sum_{n=0}^{\infty} F_{4-n}\Lambda^{4-n} a_n \quad \text{where} \quad F(\mathcal{D}_A^2) = f(\mathcal{D}_A)$$

chamseddine and connes (1996, 1997)

f : cut-off function ⇒ its taylor expansion at zero vanishes

⇒ the asymptotic expansion of the trace reduces to:

$$\boxed{\mathrm{Tr}\left(f\left(\frac{D_A}{\Lambda}\right)\right) \sim 2\Lambda^4 f_4 a_0 + 2\Lambda^2 f_2 a_2 + f_0 a_4}$$

f plays a rôle through its momenta f_0, f_2, f_4

real parameters related to the coupling constants at unification, the gravitational constant, and the cosmological constant the full lagrangian of SM, minimally coupled to gravity in euclidean form,* obtained as the asymptotic expansion (in inverse powers of Λ) of the spectral action for product ST:

chamseddine, connes, marcolli (2007)

$\star$ the discussion of phenomenological aspects relies on a wick rotation to imaginary time

$$
\begin{gathered}
\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \\
\frac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu\partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)) - igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + \\
g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H\partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \\
\beta_h\left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h - g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\frac{1}{8}g^2\alpha_h\left(H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2\right) - gMW^+_\mu W^-_\mu H - \\
\frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\,(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2+(\phi^0)^2+2\phi^+\phi^-\right) - \\
\frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-\right) - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \\
\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2s_wA_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - \\
g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^2s_w^2A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + \\
m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2-1-\gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2 + \\
\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \\
\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\,\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa - \frac{1}{4}\,\overline{\bar{\nu}_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat{\nu}_\kappa} + \\
\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \\
\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j)
\end{gathered}
$$

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu -$$

- full SM lagrangian
- majorana mass terms for right-handed neutrinos

⇒ NCSG offers a geometric interpretation of SM

- gravitational terms coupled to matter

$$ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) -$$
$$\frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-\right) - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) -$$
$$\frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) -$$
$$g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial +$$
$$m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) +$$
$$\frac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + (\bar{u}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 +$$
$$\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) +$$
$$\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \frac{g}{2}\frac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) +$$
$$\frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \frac{1}{4}\overline{\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa} +$$
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$$\frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j)$$

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$$\ldots) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu \left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) -$$

- EH action with a cosmological term
- topological term
- conformal gravity term with weyl curvature tensor
- conformal coupling of higgs to gravity

$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m_d^\lambda(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1+\gamma^5)u_j^\kappa) - m_u^\kappa(\bar d_j^\lambda C^\dagger_{\lambda\kappa}(1-\gamma^5)u_j^\kappa)\right) - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar u_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar d_j^\lambda d_j^\lambda) +$$

$$\frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u_j^\lambda\gamma^5 u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d_j^\lambda\gamma^5 d_j^\lambda)$$

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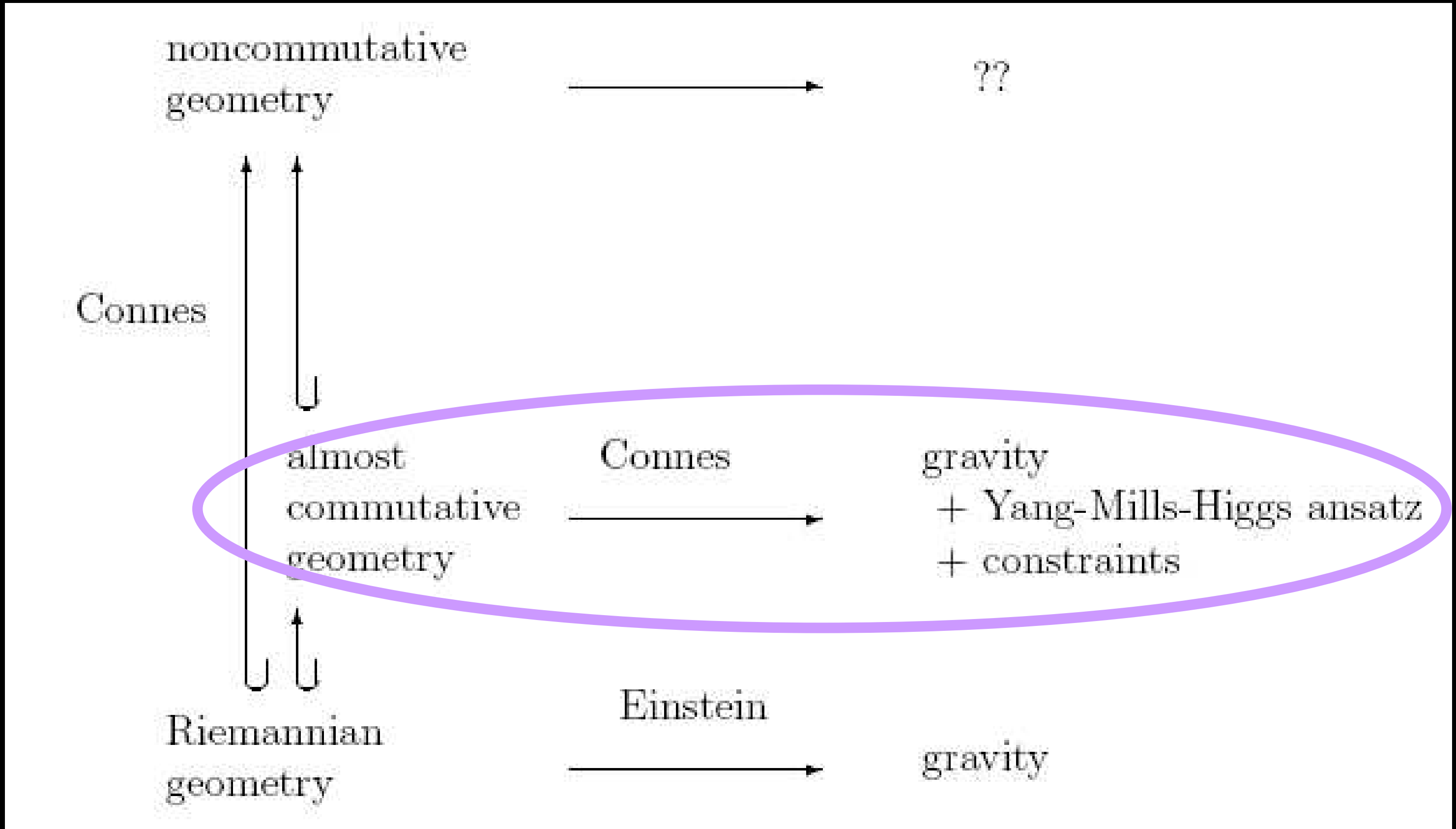
$$\frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}_j^\lambda\gamma^5 u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}_j^\lambda\gamma^5 d_j^\lambda)$$


noncommutative
geometry
??
Connes
almost
commutative
geometry
Connes
gravity
+ Yang-Mills-Higgs ansatz
+ constraints
Riemannian
geometry
Einstein
gravity

criticisms

- simple almost commutative space

  extend to less trivial noncommutative geometries

- purely classical model

  it cannot be used within EU when QC cannot be neglected

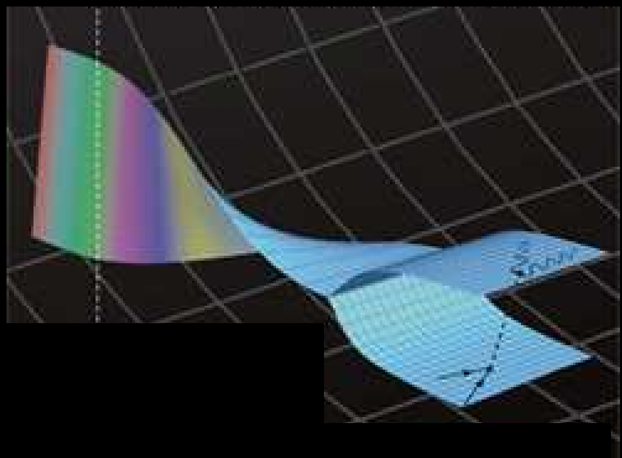
- action functional obtained through perturbative approach in inverse powers of cut-off scale

  it ceases to be valid at high energy scales

- model developed in euclidean signature

  physical studies must be done in lorentzian signature

doubling of the algebra



$\mathcal{M}$ $\quad (C^\infty(\mathcal{M}), L^2(\mathcal{M}, S), \partial\!\!\!/_{\mathcal{M}})$

$\mathcal{F}$ $\quad (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}}, J_{\mathcal{F}}, \gamma_{\mathcal{F}})$

$\mathcal{M} \times \mathcal{F}$

$$(\mathcal{A}, \mathcal{H}, \mathcal{D}, J, \gamma) = (C^\infty(\mathcal{M}), L^2(\mathcal{M}, S), \partial\!\!\!/_{\mathcal{M}}, J_{\mathcal{M}}, \gamma_5) \otimes (\mathcal{A}_{\mathcal{F}}, \mathcal{H}_{\mathcal{F}}, D_{\mathcal{F}}, J_{\mathcal{F}}, \gamma_{\mathcal{F}})$$

defined as $(\mathcal{A}, \mathcal{H}, \mathcal{D}, J, \gamma) = (\mathcal{A}_1, \mathcal{H}_1, \mathcal{D}_1, J_1, \gamma_1) \otimes (\mathcal{A}_2, \mathcal{H}_2, \mathcal{D}_2, J_2, \gamma_2)$

with

$$\mathcal{A} = \mathcal{A}_1 \otimes \mathcal{A}_2 \ , \ \ \mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \ ,$$
$$\mathcal{D} = \mathcal{D}_1 \otimes 1 + \gamma_1 \otimes \mathcal{D}_2 \ ,$$
$$\gamma = \gamma_1 \otimes \gamma_2 \ , \quad J = J_1 \otimes J_2 \ ,$$

$$J^2 = -1, \quad [J, \mathcal{D}] = 0,$$

$$[J_1, \gamma_1] = 0, \quad \{J, \gamma\} = 0,$$

the doubling of the algebra is related to dissipation and the gauge field structure

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

brownian motion:

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

derived from a delta functional classical constraint representation as a functional integral

$$\delta[m\ddot{x} + \gamma\dot{x} - f] = - \int \mathcal{D}y \ \exp\left[\frac{i}{\hbar}\int dt\,[m\ddot{x} + \gamma\dot{x} - f]\right.$$

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

brownian motion:

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

the constraint condition at the classical level introduces a new coordinate y

derived from a delta functional classical constraint representation as a functional integral

$$\delta[m\ddot{x} + \gamma\dot{x} - f] = - \int \mathcal{D}y \, \exp\left[\frac{i}{\hbar}\int dt\,[m\ddot{x} + \gamma\dot{x} - f]\right]$$

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the doubling of the algebra is related to dissipation and the gauge field structure

$$\frac{d}{dt}\frac{\partial L_f}{\partial \dot{y}} = \frac{\partial L_f}{\partial y} \; ; \qquad \frac{d}{dt}\frac{\partial L_f}{\partial \dot{x}} = \frac{\partial L_f}{\partial x}$$

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

$$\Longrightarrow \quad \boxed{m\ddot{x} + \gamma\dot{x} = f \; , \qquad m\ddot{y} - \gamma\dot{y} = 0}$$

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

canonical formalism for dissipative systems

x-system: open (dissipating) system

$$m\ddot{x}(t) + \gamma\dot{x}(t) = f(t)$$

$$\frac{d}{dt}\frac{\partial L_f}{\partial \dot{y}} = \frac{\partial L_f}{\partial y} \; ; \qquad \frac{d}{dt}\frac{\partial L_f}{\partial \dot{x}} = \frac{\partial L_f}{\partial x}$$

$$L_f(\dot{x}, \dot{y}, x, y) = m\dot{x}\dot{y} + \frac{\gamma}{2}(x\dot{y} - y\dot{x}) + fy$$

$$\boxed{m\ddot{x} + \gamma\dot{x} = f \; , \qquad m\ddot{y} - \gamma\dot{y} = 0}$$

{x – y} is a closed system

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

$$m\ddot{x} + \gamma\dot{x} + kx = 0$$

$$m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$x_1(t) = \frac{x(t) + y(t)}{\sqrt{2}} , \qquad x_2(t) = \frac{x(t) - y(t)}{\sqrt{2}}$$

$$A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i, j = 1, 2)$$

$$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2)$$

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

the doubling of the algebra is related to dissipation and the gauge field structure

$$\boxed{L = \frac{m}{2}(\dot{x}_1^2 - \dot{x}_2^2) + \frac{e}{2}(\dot{x}_1 A_1 + \dot{x}_2 A_2) - e\Phi}$$

$$\boxed{A_i = \frac{B}{2}\epsilon_{ij}x_j \quad (i, j = 1, 2)}$$

$$\Phi \equiv (k/2/e)({x_1}^2 - {x_2}^2)$$

- doubled coordinate, e.g. x_2 acts as gauge field component A_1 to which x_1 coordinate is coupled
- energy dissipated by one system is gained by the other one
- gauge field as *bath/reservoir* in which the system is embedded

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

dissipation, may lead to a quantum evolution

't hooft's conjecture: loss of information (dissipation) in a regime of deterministic dynamics may lead to QM evolution

sakellariadou, stabile, vitiello , PRD 84 (2011) 045026

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$$m\ddot{x} + \gamma\dot{x} + kx = 0 \qquad m\ddot{y} - \gamma\dot{y} + ky = 0$$

$$H = H_{\mathrm{I}} - H_{\mathrm{II}} \qquad H_{\mathrm{II}}|\psi\rangle = 0 \implies \text{info loss}$$

to define physical states and guarantie that H is bounded from below

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

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$$H = H_{\mathrm{I}} - H_{\mathrm{II}} \qquad H_{\mathrm{II}}|\psi\rangle = 0 \Longrightarrow \text{info loss}$$

to define physical states and guarantie that H is bounded from below

physical states are invariant under time reversal and periodical (τ)

$$_H\langle\psi(\tau)|\psi(0)\rangle_H = e^{i\phi} = e^{i\alpha\pi}$$

$$\boxed{\langle\psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega(n + \tfrac{\alpha}{2}) = \hbar\Omega n + E_0}$$

sakellariadou, stabile, vitiello, PRD 84 (2011) 045026

dissipation, may lead to a quantum evolution

dissipation term in H of classical damped-amplified oscillators manifests itself as geometric phase and leads to zero point energy

$$ {}_H\langle\psi(\tau)|\psi(0)\rangle_H = e^{i\phi} = e^{i\alpha\pi} $$

$$ \boxed{\langle\psi_n(\tau)|H|\psi_n(\tau)\rangle = \hbar\Omega(n+\tfrac{\alpha}{2}) = \hbar\Omega n + E_0} $$

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algebra doubling → *deformed hopf algebra*

- define coproduct operators
- build bogogliubov operators as linear combinations of coproduct ones

⇒ neutrino oscillations

gargiulo, sakellariadou, vitiello; EPJC (2014)

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$\Rightarrow$ neutrino oscillations

mixing transformations connecting flavor fields ψ_f to fields ψ_m

$$\nu_e(x) = G_\theta^{-1}(t)\nu_1(x)G_\theta(t) \; ,$$
$$\nu_\mu(x) = G_\theta^{-1}(t)\nu_2(x)G_\theta(t) \; .$$

generator of field mixing transf. $G_\theta(t)$

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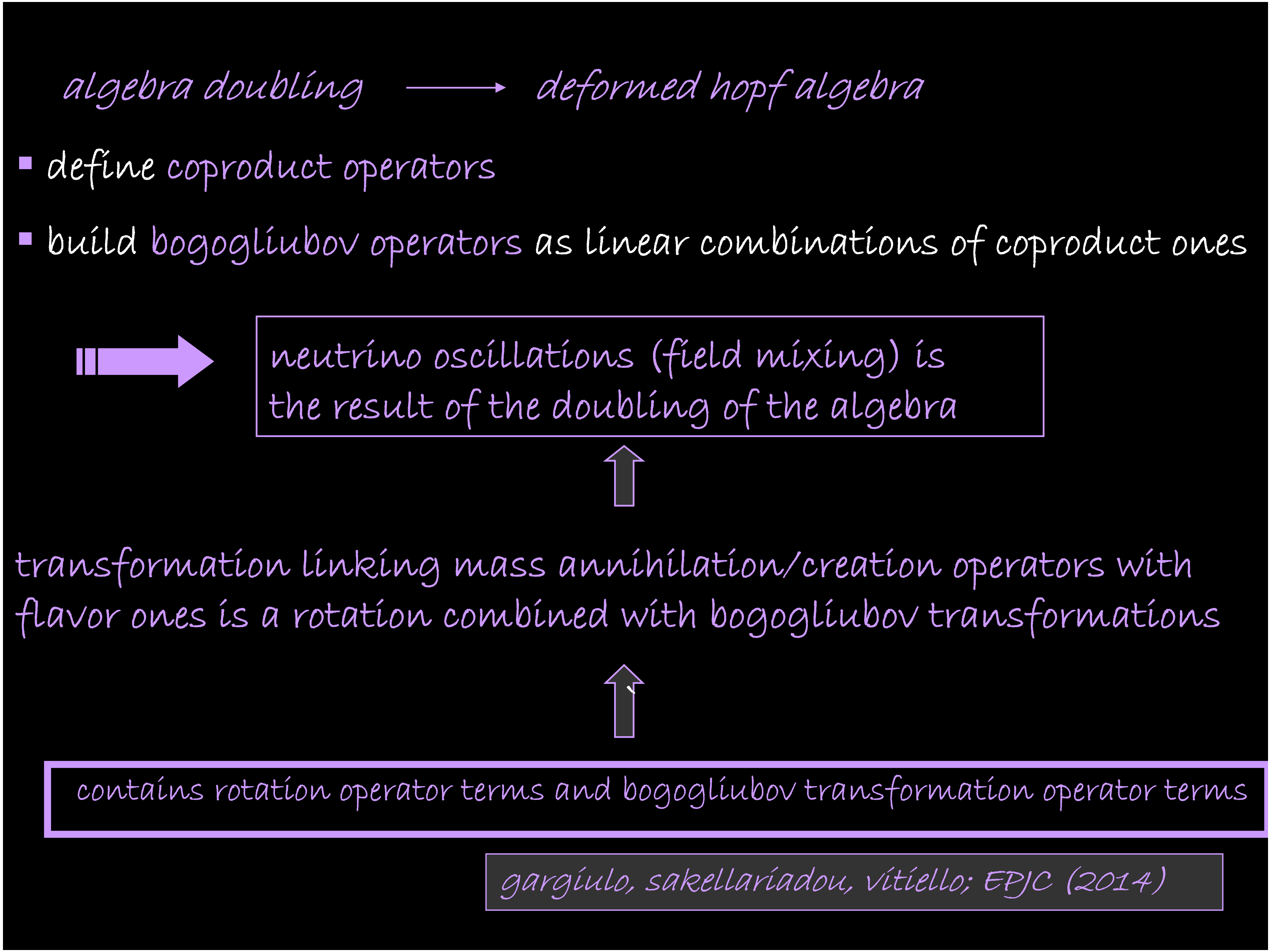
generator of field mixing transf. $G_\theta(t)$

contains rotation operator terms and bogogliubov transformation operator terms

gargiulo, sakellariadou, vitiello; EPJC (2014)


algebra doubling → deformed hopf algebra
▪ define coproduct operators
▪ build bogogliubov operators as linear combinations of coproduct ones
neutrino oscillations (field mixing) is the result of the doubling of the algebra
transformation linking mass annihilation/creation operators with flavor ones is a rotation combined with bogogliubov transformations
contains rotation operator terms and bogogliubov transformation operator terms
gargiulo, sakellariadou, vitiello; EPJC (2014)

phenomenology

- algebra $\mathcal{A}_{\mathcal{F}}$ of the discrete space $\mathcal{F}$: $M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$

$4^2 = 16$ fermions (# of states on Hilbert space) per family

- gauge bosons: inner fluctuations along continuous directions

- Higgs doublet: inner fluctuations along discrete directions

- mass of the Higgs doublet with –tive sign and a quartic term with a + sign $\Rightarrow$ mechanism for SSB of EW symmetry

- assuming f is approximated by cut-off function

$$Tr(f(D/\Lambda))$$

normalisation of kinetic terms:

$$\boxed{\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4}}$$

$$\boxed{g_2^2 = g_3^2 = \frac{5}{3} g_1^2}$$

coincide with those obtained in GUTs

$$\Rightarrow \quad \boxed{\sin^2 \theta_W = \frac{3}{8}}$$

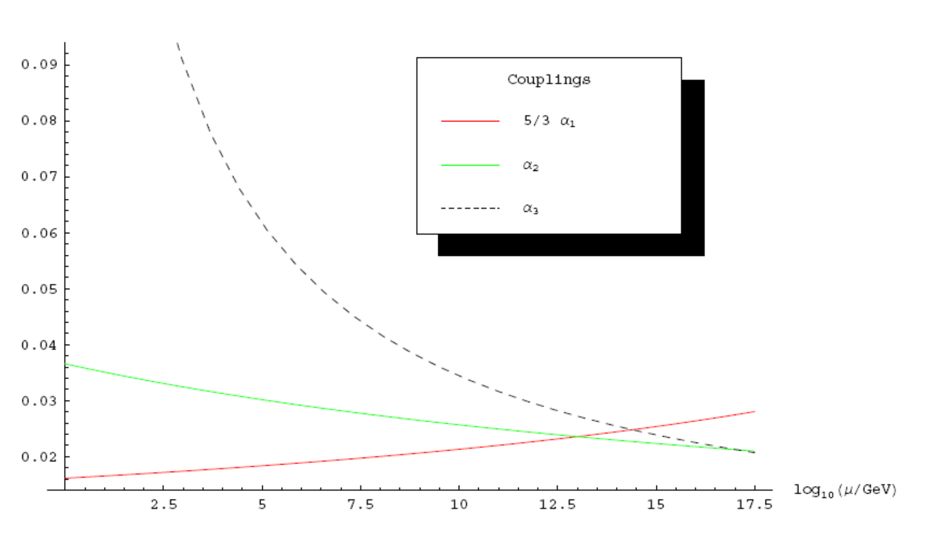
a value also obtained in SU(5) and SO(10)

chamseddine, connes, marcolli (2007)

- assuming big desert hypothesis, the running of the couplings $\alpha_i = g_i^2/(4\pi)$, $i = 1, 2, 3$ up to 1-loop corrections:

$$\beta_i = \frac{1}{(4\pi)^2} b_i g_i^3 \quad \text{with} \quad b = \left(\frac{41}{5}, -\frac{19}{6}, -7\right)$$

not a unique unification energy



- big desert hypothesis approximately valid
- f can be approximated by the cut-off function but there are small deviations

chamseddine, connes, marcolli (2007)

- see-saw mechanism for m_ν with large $m_{\nu_{\text{right-handed}}}$

- constraint on yukawa couplings at unification scale:

$$\sum_\sigma (y_\nu^\sigma)^2 + (y_e^\sigma)^2 + 3(y_u^\sigma)^2 + +3(y_d^\sigma)^2 = 4g^2$$

- mass of top quark:

at unification scale $\boxed{m_{\text{top}} \sim 179 \ \text{GeV}}$, the $g \sim 0.517$
the RGE predicts $\Lambda \sim 1.1 \times 10^{17}\text{GeV}$

chamseddine, connes, marcolli (2007)

higgs field as a gauge field associated to the finite space

- mass of the higgs:

a distinctive feature of the spectral action is that the <u>higgs coupling is proportional to the gauge couplings</u>

⇒ a restriction on its mass

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real scalar singlet is associated with the majorana mass of right-handed neutrino; it is nontrivally mixed with higgs

connes, chamseddine (2012)

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$$m_{\text{higgs}} \sim 170 \text{ GeV}$$

connes, chamseddine (2012)

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$$m_{\text{higgs}} \sim 170 \ \text{GeV}$$

new approach: higgs doublet and singlet get mixed

⇒ masses of higgs and singlet get shifted

⇒ consistency with 125 GeV for higgs mass and 170 GeV for top quark mass

connes, chamseddine (2012)

other attempts

- build a model based on a larger symmetry

$$\mathcal{A} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C}) \qquad k = 2a$$

up to now $a = 2$

consider $a = 4$ $\quad \mathcal{A}_G = M_4(\mathbb{H}) \oplus M_8(\mathbb{C})$

it explains the presence of the field necessary for a correct fit of the mass of the higgs around 126 GeV

devastato, lizzi, martinetti (2013)

- generalise inner fluctuations to real spectral triples that fail on the first order condition (i.e., the dirac operator is a differential operator of order 1):

$$[[D,a],JbJ^{-1}]=0\,,\qquad \forall a,b\in\mathcal{A}.$$

antilinear isometry $J:\mathcal{H}\to\mathcal{H}$

so far, the fluctuated metrics are of the form:

$$D'=D+A+\epsilon JAJ^{-1},\quad A=\sum a_j[D,b_j]$$

$\epsilon\in\{\pm 1\}$ $\quad JDJ^{-1}=\epsilon D$

pati-salam type model $SU(2)_R\times SU(2)_L\times SU(4)$

chamseddine, connes, van suijlekom (2013)

comment

thermal history of the universe

$$\mathrm{G_{SM}} = \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$$

$$\underset{\mathrm{GUT}}{G} \to H_1 \to H_2 \to \cdots \to G_{\mathrm{SM}}$$

comment

thermal history of the universe

$$\mathrm{G_{SM}} = \mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$$

$$\underset{\mathrm{GUT}}{G} \to H_1 \to H_2 \to \cdots \to G_{\mathrm{SM}}$$

study homotopy $\pi_k(\mathcal{M}_n)$ group of false vacuum $\mathcal{M}_n = G/H$

$\pi_k(\mathcal{M}_n) \neq 0$ ⇨ topological defects

k=0 ⇨ domain walls

k=1 ⇨ cosmic strings

k=2 ⇨ monopoles

$$
\mathrm{4_C\ 2_L\ 2_R}\ \left\{\begin{array}{lll}
\xrightarrow{1} \ \mathrm{3_C\ 2_L\ 2_R\ 1_{B-L}} & \left\{\begin{array}{l} \xrightarrow{1} \ \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} \ \xrightarrow{2\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{2'\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \end{array}\right. \\
\xrightarrow{1} \ \mathrm{4_C\ 2_L\ 1_R} & \left\{\begin{array}{l} \xrightarrow{1} \ \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} \ \xrightarrow{2\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{2'\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \end{array}\right. \\
\xrightarrow{1} \ \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} & \xrightarrow{2\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \\
\xrightarrow{1\ (1,2)} \ \mathrm{G_{SM}}\ (Z_2) &
\end{array}\right.
$$

$$
\mathrm{4_C\ 2_L\ 2_R}\ Z_2^{\mathrm{C}}\ \left\{\begin{array}{lll}
\xrightarrow{1} \ \mathrm{3_C\ 2_L\ 2_R\ 1_{B-L}}\ Z_2^{\mathrm{C}} & \left\{\begin{array}{l} \xrightarrow{3} \ \mathrm{3_C\ 2_L\ 2_R\ 1_{B-L}} \ \longrightarrow \ \cdots \\ \xrightarrow{1,3} \ \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} \ \xrightarrow{2\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{2',3\ (2,3)} \ \mathrm{G_{SM}}\ (Z_2) \end{array}\right. \\
\xrightarrow{1} \ \mathrm{4_C\ 2_L\ 1_R}\ Z_2^{\mathrm{C}} & \left\{\begin{array}{l} \xrightarrow{3} \ \mathrm{4_C\ 2_L\ 1_R} \ \longrightarrow \ \cdots \\ \xrightarrow{1,3} \ \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} \ \xrightarrow{2\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \\ \xrightarrow{3\ (2,3)} \ \mathrm{G_{SM}}\ (Z_2) \end{array}\right. \\
\xrightarrow{3} \ \mathrm{4_C\ 2_L\ 2_R} & \longrightarrow \ \text{Eq. (4.10)} \\
\xrightarrow{1} \ \mathrm{4_C\ 2_L\ 1_R} & \longrightarrow \ \cdots \\
\xrightarrow{1,3} \ \mathrm{3_C\ 2_L\ 2_R\ 1_{B-L}} & \longrightarrow \ \cdots \\
\xrightarrow{1,3} \ \mathrm{3_C\ 2_L\ 1_R\ 1_{B-L}} & \xrightarrow{2\ (2)} \ \mathrm{G_{SM}}\ (Z_2) \\
\xrightarrow{1,3\ (1,2,3)} \ \mathrm{G_{SM}}\ (Z_2). &
\end{array}\right.
$$

jeannerot, rocher, sakellariadou, PRD 68 (2003) 103514

$$4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{2'\ (2)} {\rm G}_{\rm SM}\ (Z_2) \end{cases} \\ \xrightarrow{1} 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{2'\ (2)} {\rm G}_{\rm SM}\ (Z_2) \end{cases} \\ \xrightarrow{1} 3_{\rm C} \ldots \xrightarrow{2\ (2)} \ldots \\ \xrightarrow{1\ (1,2)} {\rm G}_{\rm S}\ldots \end{cases}$$

$$G_{\rm GUT} \xrightarrow{M_{\rm GUT}} H_1 \xrightarrow[\Phi_+\Phi_-]{M_{\rm infl}} H_2 \longrightarrow G_{\rm SM}$$

$$4_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ Z_2^{\rm C} \begin{cases} \xrightarrow{1} 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L}\ Z_2^{\rm C} \begin{cases} \ldots \longrightarrow \cdots \\ \xrightarrow{1,3} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{2',3\ (2,3)} {\rm G}_{\rm SM}\ (Z_2) \end{cases} \\ \xrightarrow{1} 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ Z_2^{\rm C} \begin{cases} \xrightarrow{3} 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} \longrightarrow \cdots \\ \xrightarrow{1,3} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{3\ (2,3)} {\rm G}_{\rm SM}\ (Z_2) \end{cases} \\ \xrightarrow{3} 4_{\rm C}\ 2_{\rm L}\ 2_{\rm R} \longrightarrow {\rm Eq.}\ (4.10) \\ \xrightarrow{1} 4_{\rm C}\ 2_{\rm L}\ 1_{\rm R} \longrightarrow \cdots \\ \xrightarrow{1,3} 3_{\rm C}\ 2_{\rm L}\ 2_{\rm R}\ 1_{\rm B-L} \longrightarrow \cdots \\ \xrightarrow{1,3} 3_{\rm C}\ 2_{\rm L}\ 1_{\rm R}\ 1_{\rm B-L} \xrightarrow{2\ (2)} {\rm G}_{\rm SM}\ (Z_2) \\ \xrightarrow{1,3\ (1,2,3)} {\rm G}_{\rm SM}\ (Z_2). \end{cases}$$

jeannerot, rocher, sakellariadou, PRD 68 (2003) 103514

$$\mathrm{SO}(10) \to \cdots \to \mathrm{G}_{3,2,2,\mathrm{B-L}} \to \mathrm{G}_{\mathrm{SM}} \times Z_2 \to \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{Q}} \times Z_2$$
$$\mathrm{SO}(10) \to \cdots \to \mathrm{G}_{3,2,1,\mathrm{B-L}} \to \mathrm{G}_{\mathrm{SM}} \times Z_2 \to \mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{U}(1)_{\mathrm{Q}} \times Z_2$$

none of the singlets of SM symmetries in minimal set of SO(10) rep. can satisfy conditions for scalar field to be inflaton

cacciapaglia, sakellariadou, arXiv:1306.3242

cosmological consequences

$$\mathcal{S}^{\mathrm{E}}_{\text{bosonic}} = \int \left(\frac{1}{2\kappa_0^2} R + \alpha_0 C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \gamma_0 + \tau_0 R^{\star} R^{\star} + \frac{1}{4} G^i_{\mu\nu} G^{\mu\nu i} + \frac{1}{4} F^{\alpha}_{\mu\nu} F^{\mu\nu\alpha} + \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} |D_\mu \mathbf{H}|^2 - \mu_0^2 |\mathbf{H}|^2 - \xi_0 R |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4 \right) \sqrt{g}\, d^4x \,,$$

bare action a la wislon

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}} \,,$$

$$\alpha_0 = -\frac{3 f_0}{10\pi^2} \,,$$

$$\gamma_0 = \frac{1}{\pi^2} \left(48 f_4 \Lambda^4 - f_2 \Lambda^2 \mathfrak{c} + \frac{f_0}{4} \mathfrak{d} \right) \,,$$

$$\tau_0 = \frac{11 f_0}{60\pi^2} \,,$$

$$\mu_0^2 = 2\Lambda^2 \frac{f_2}{f_0} - \frac{\mathfrak{e}}{\mathfrak{a}} \,,$$

$$\xi_0 = \frac{1}{12} \,,$$

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2} \,;$$

$$\mathbf{H} = (\sqrt{\mathfrak{a} f_0}/\pi)\phi$$

$\mathfrak{a}, \mathfrak{b}, \mathfrak{c}, \mathfrak{d}, \mathfrak{e}$ describe possible choices of $D_{\mathcal{F}}$

yukawa parameters and majorana terms for ν_{R}

comment

comment

one may wonder whether the quadratic curvature terms indicate the emergence of negative energy massive graviton modes

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the higher derivative terms that are quadratic in curvature lead to:

$$\int \Big(\frac{1}{2\eta} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{\omega}{3\eta} R^2 + \frac{\theta}{\eta} E\Big) \sqrt{-\,g}\, d^4x$$

$$E = R^* R^*$$

topological term; the integrand in the Euler characteristic

$$\int E\sqrt{-g}d^4x = \int R^\star R^\star \sqrt{-g}d^4x$$

comment

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the higher derivative terms that are quadratic in curvature lead to:

$$\int \left(\frac{1}{2\eta} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} \quad - \frac{\omega}{3\eta} R^2 + \frac{\theta}{\eta} E\right) \sqrt{-\,g}\, d^4x$$

$E = R^* R^*$ topological term; the integrand in the Euler characteristic

the running of the coefficients of the higher derivative terms is determined by RGE

to avoid low energy constraints, the coefficients of the quadratic curvature terms $R_{\mu\nu}R^{\mu\nu}$, R^2 should not exceed 10^{74}; indeed the case

corrections to einstein's equations

gravitational & coupling between Higgs field and Ricci curvature

⟹ equations of motion

nelson, sakellariadou, PRD 81 (2010) 085038

gravitational & coupling between Higgs field and Ricci curvature

⇒ equations of motion

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{\beta^2}\delta_{\rm cc}\left[2C^{\mu\lambda\nu\kappa}_{\ ;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2 \delta_{\rm cc} T^{\mu\nu}_{\rm matter}$$

$$\beta^2 \equiv -\frac{1}{4\kappa_0^2\alpha_0}$$

$$\delta_{\rm cc} \equiv [1 - 2\kappa_0^2 \xi_0 \mathbf{H}^2]^{-1}$$

$$\alpha_0 = \frac{-3f_0}{10\pi^2}$$

nelson, sakellariadou, PRD 81 (2010) 085038

gravitational & coupling between Higgs field and Ricci curvature

$\Rightarrow$ equations of motion

neglect nonminimal coupling between geometry and higgs

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{1}{\beta^2}\delta_{cc}\left[2C^{\mu\lambda\nu\kappa}_{\;;\lambda;\kappa} + C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\right] = \kappa_0^2\delta_{cc}T^{\mu\nu}_{\text{matter}}$$

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FLRW

weyl tensor vanishes

NCSG corrections to einstein equations vanish

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gravitational & coupling between Higgs field and Ricci curvature

⟹ equations of motion

neglect nonminimal coupling between geometry and higgs

⟹ corrections to einstein's eqs. will be apparent at leading order, only in anisotropic models

bianchi model: NCSG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic spacetimes

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$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

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$$g_{\mu\nu} = \mathrm{diag}\left[-1, \{a_1(t)\}^2 e^{-2nz}, \{a_2(t)\}^2 e^{-2nz}, \{a_3(t)\}^2\right]$$

$$A_i(t) = \ln a_i(t)$$

$$\begin{aligned}
\kappa_0^2 T_{00} = {} & \\
& -\dot{A}_3\left(\dot{A}_1+\dot{A}_2\right) - n^2 e^{-2A_3}\left(\dot{A}_1\dot{A}_2 - 3\right) \\
& +\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\Big[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2 \\
& -\dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3\Big] \\
& -\frac{4\alpha_0\kappa_0^2}{3}\sum_i\Big\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i \\
& +\dot{A}_i\dot{A}_{i+1}\left(\left(\dot{A}_i-\dot{A}_{i+1}\right)^2 - \dot{A}_i\dot{A}_{i+1}\right) \\
& +\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\Big[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1}+\ddot{A}_{i+2}\right) \\
& +\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\Big] \\
& +\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right] \\
& \times\left[2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right]\Big\}
\end{aligned}$$

nelson, sakellariadou, PRD 81 (2010) 085038

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$$+\frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}\left[5\left(\dot{A}_1\right)^2 + 5\left(\dot{A}_2\right)^2 - \left(\dot{A}_3\right)^2\right.$$

neglecting nonminimal coupling between geometry and higgs field, NCG corrections to einstein's eqs. are present only in inhomogeneous and anisotropic space-times

$$A_i(t) = \ln a_i(t)$$

$$+\left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left[-\ddot{A}_i - \left(\dot{A}_i\right)^2 + \frac{1}{2}\left(\ddot{A}_{i+1} + \ddot{A}_{i+2}\right)\right.$$

$$\left.+\frac{1}{2}\left(\left(\dot{A}_{i+1}\right)^2 + \left(\dot{A}_{i+2}\right)^2\right)\right]$$

$$+\left[\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - \left(\ddot{A}_i + \left(\dot{A}_i\right)^2\right)\left(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}\right)\right]$$

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nelson, sakellariadou, PRD <u>81</u> (2010) 085038

at energies approaching higgs scale, the nonminimal coupling of higgs field to curvature cannot be neglected

nelson, sakellariadou, PRD 81 (2010) 085038

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$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \left[\frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2 / 6} \right] T^{\mu\nu}_{\mathrm{matter}}$$

$\Longrightarrow$ effective gravitational constant

nelson, sakellariadou, PRD 81 (2010) 085038

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$\Longrightarrow$ effective gravitational constant

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12}|\mathbf{H}|^2 + \frac{1}{2}|D^\alpha \mathbf{H}||D^\beta \mathbf{H}|g_{\alpha\beta} - \mu_0|\mathbf{H}|^2 + \lambda_0|\mathbf{H}|^4$$

$$\Longrightarrow \quad -\mu_0|\mathbf{H}|^2 \to -\left(\mu_0 + \frac{R}{12}\right)|\mathbf{H}|^2$$

$\Longrightarrow$ increases the higgs mass

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remarks

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- redefine higgs: $\tilde{\phi} = -\ln\left(|\mathbf{H}|/(2\sqrt{3})\right)$

⇒ *rewrite higgs lagrangian in terms of 4dim dilatonic gravity*

$$\boxed{\mathcal{L}_{\tilde{\phi}} = e^{-2\tilde{\phi}}\left[-R + 6D^{\alpha}\tilde{\phi}D^{\beta}\tilde{\phi}g_{\alpha\beta} - 12\left(\mu_0 - 12\lambda_0 e^{-2\tilde{\phi}}\right)\right]}$$

link with compactified string models

remarks

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link with compactified string models

- chameleon models
  scalar field with nonminimal coupling to standard matter

  NCSG
  scalar field (higgs) with nonzero coupling to bckg geometry
  mass & dynamics of higgs dependent on local matter content

link with chameleon cosmology

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$$\mathbf{H} = (\sqrt{a f_0}/\pi)\phi$$

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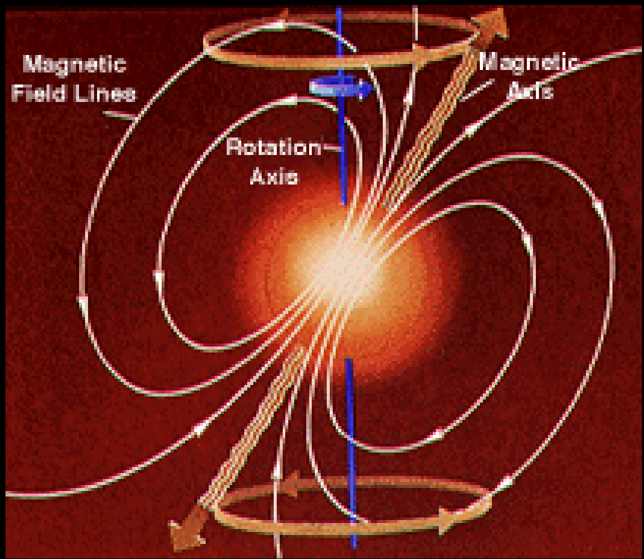
yukawa parameters and majorana terms for ν_R

gravitational waves in NCSG



nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

linear perturbations around minkowski background in synchronous gauge:

$$g_{\mu\nu} = \mathrm{diag}\left(\{a(t)\}^2\left[-1, \left(\delta_{ij} + h_{ij}\left(x\right)\right)\right]\right)$$

$a(t) = 1$ $\nabla_i h^{ij} = 0$

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

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$$\boxed{\left(\Box - \beta^2\right)\Box h^{\mu\nu} = \beta^2 \frac{16\pi G}{c^4} T^{\mu\nu}_{\mathrm{matter}}}$$

with conservation eqs: $\frac{\partial}{\partial x^\mu} T^\mu_{\ \nu} = 0$

$$\beta^2 = -\frac{1}{32\pi G \alpha_0} \qquad \alpha_0 = \frac{-3f_0}{10\pi^2}$$

$$\frac{g_3^2 f_0}{2\pi^2} = \frac{1}{4} \qquad g_3^2 = g_2^2 = \frac{5}{3} g_1^2$$

nelson, ochoa, sakellariadou, PRD 82 (2010) 085021

energy lost to gravitational radiation by orbiting binaries:

$$\boxed{-\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} \approx \frac{c^2}{20G}|\mathbf{r}|^2 \dot{h}_{ij}\dot{h}^{ij}}$$

strong deviations from GR at frequency scale

$$2\omega_c \equiv \beta c \sim (f_0 G)^{-1/2} c$$

set by the moments of the test function f

scale at which NCSG effects become dominant

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restrict β by requiring that the magnitude of deviations from GR must by less than the uncertainty

Binary	Distance (pc)	Orbital Period (hr)	Eccentricity	GR (%)
PSR J0737-3039	~ 500	2.454	0.088	0.2
PSR J1012-5307	~ 840	14.5	$< 10^{-6}$	10
PSR J1141-6545	> 3700	4.74	0.17	6
PSR B1916+16	~ 6400	7.752	0.617	0.1
PSR B1534+12	~ 1100	10.1	?	1
PSR B2127+11C	~ 9980	8.045	0.68	3

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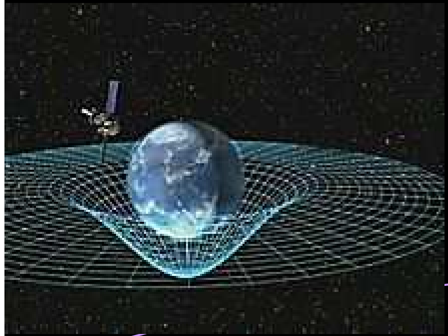
$$\beta > 7.55 \times 10^{-13}\mathrm{m}^{-1}$$

nelson, ochoa, sakellariadou, PRL 105 (2010) 101602

accuracy to which the rate of change of orbital period agrees with predictions of GR

gravity probe B

the satellite contains a set of gyroscopes in low circular polar orbit with altitude h=650 km



geodesic precession in the orbital plane
lense-thirring (frame dragging) precession in the plane of earth equator

GR

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

milliarcsec/yr

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

e.o.m. for gyro spin 3 vector S:

$$\frac{d\mathbf{S}}{dt} = \left.\frac{d\mathbf{S}}{dt}\right|_{\mathrm{G}} + \left.\frac{d\mathbf{S}}{dt}\right|_{\mathrm{LT}}$$

metric:

$$ds^2 = -(1+2\Phi)dt^2 + 2\mathbf{A}\cdot d\mathbf{x}dt + (1+2\Psi)d\mathbf{x}^2$$

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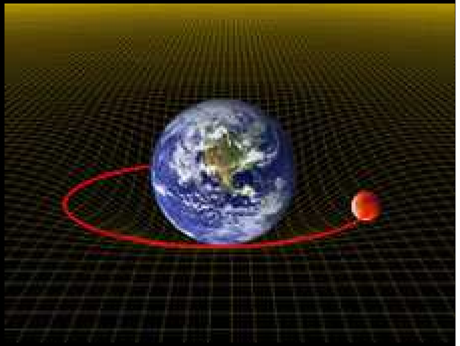
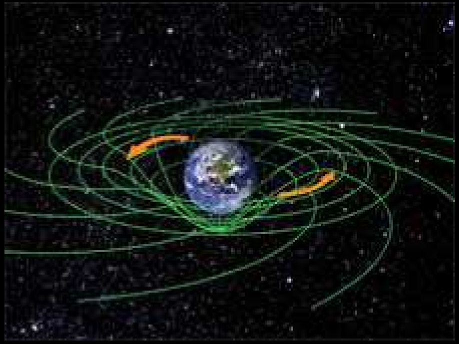
instantaneous geodesic precession



$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} = \mathbf{\Omega}_{\mathrm{G}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{G}} = \frac{1}{2}[\nabla(\Phi - 2\Psi)] \wedge \mathbf{v}$$

instantaneous lense-thirring precession



$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{LT}} = \mathbf{\Omega}_{\mathrm{LT}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{LT}} = \frac{1}{2}\nabla \wedge \mathbf{A}$$

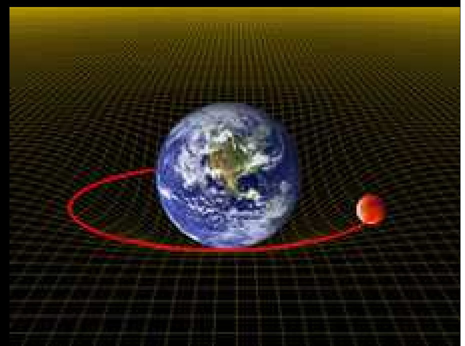
e.o.m. for gyro spin 3 vector S:

$$\beta \geq 10^{-6}\ \mathrm{m}^{-1}$$

$$\frac{d\mathbf{S}}{dt} = \frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} + \frac{d\mathbf{S}}{dt}\Big|_{\mathrm{LT}}$$

metric: $$ds^2 = -(1+2\Phi)dt^2 + 2\mathbf{A}\cdot d\mathbf{x}dt + (1+2\Psi)d\mathbf{x}^2$$

instantaneous geodesic precession



$$\frac{d\mathbf{S}}{dt}\Big|_{\mathrm{G}} = \mathbf{\Omega}_{\mathrm{G}} \wedge \mathbf{S} \quad \text{with} \quad \mathbf{\Omega}_{\mathrm{G}} = \frac{1}{2}[\nabla(\Phi - 2\Psi)] \wedge \mathbf{v}$$

$$\Omega_{\text{geodesic}} = \Omega_{\text{geodesic(GR)}} + \Omega_{\text{geodesic(NCG)}}$$

Effect	Measured	Predicted
Geodesic precession	6602 ± 18	6606
Lense-Thirring precession	37.2 ± 7.2	39.2

$$|\Omega_{\text{geodesic(NCG)}}| \leq \delta\Omega_{\text{geodesic}}$$

$$\delta\Omega_{\text{geodesic}} = 18\ \ \text{mas/y}$$

constraints from torsion balance

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

constraints from torsion balance

the modifications induced by NCSG action to the newtonian potentials Φ, Ψ are similar to those induced by a fifth-force through a potential

$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

constraints from torsion balance

the modifications induced by NCSG action to the newtonian potentials Φ, Ψ are similar to those induced by a fifth-force through a potential

$$V(r) = -\frac{GMm}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

$$\lambda = \beta^{-1} \qquad \alpha \sim \mathcal{O}(1)$$

a more stringent constraint on β can be obtained using the results from laboratory experiments design to test the fifth force

$$\lambda \lesssim 10^{-4}\mathrm{m} \quad \Longrightarrow \quad \beta \geq 10^4 \ \mathrm{m}^{-1}$$

eot-wash and irvine experiments

lambiase, sakellariadou, stabile, JCAP 12 (2013) 020

inflation through the nonminimal coupling between the geometry and the higgs field

inflation through the nonminimal coupling between the geometry and the higgs field

<u>proposal</u>: the higgs field, could play the rôle of the inflaton

but

GR: to get the amplitude of density perturbations, the higgs mass would have to be 11 orders of magnitude **higher**

re-examine the validity of this statement within NCSG

nelson, sakellariadou, PLB <u>680</u> (2009) 263

buck, fairbairn, sakellariadou, PRD <u>82</u> (2010) 043509

$$S^{\mathrm{L}}_{\mathrm{GH}} = \int \Big[\frac{1 - 2\kappa_0^2 \xi_0 H^2}{2\kappa_0^2} R - \frac{1}{2}(\nabla H)^2 - V(H)\Big]\sqrt{-g}\; d^4x$$

$$V(H) = \lambda_0 H^4 - \mu_0^2 H^2$$

subject to radiative corrections as a function of energy

$$\kappa_0^2 = \frac{12\pi^2}{96 f_2 \Lambda^2 - f_0 \mathfrak{c}}$$

$$f_0 = \pi^2/(2g^2)$$

$$\xi_0 = \frac{1}{12}$$

a priori unconstrained

$$\lambda_0 = \frac{\pi^2 \mathfrak{b}}{2 f_0 \mathfrak{a}^2}$$

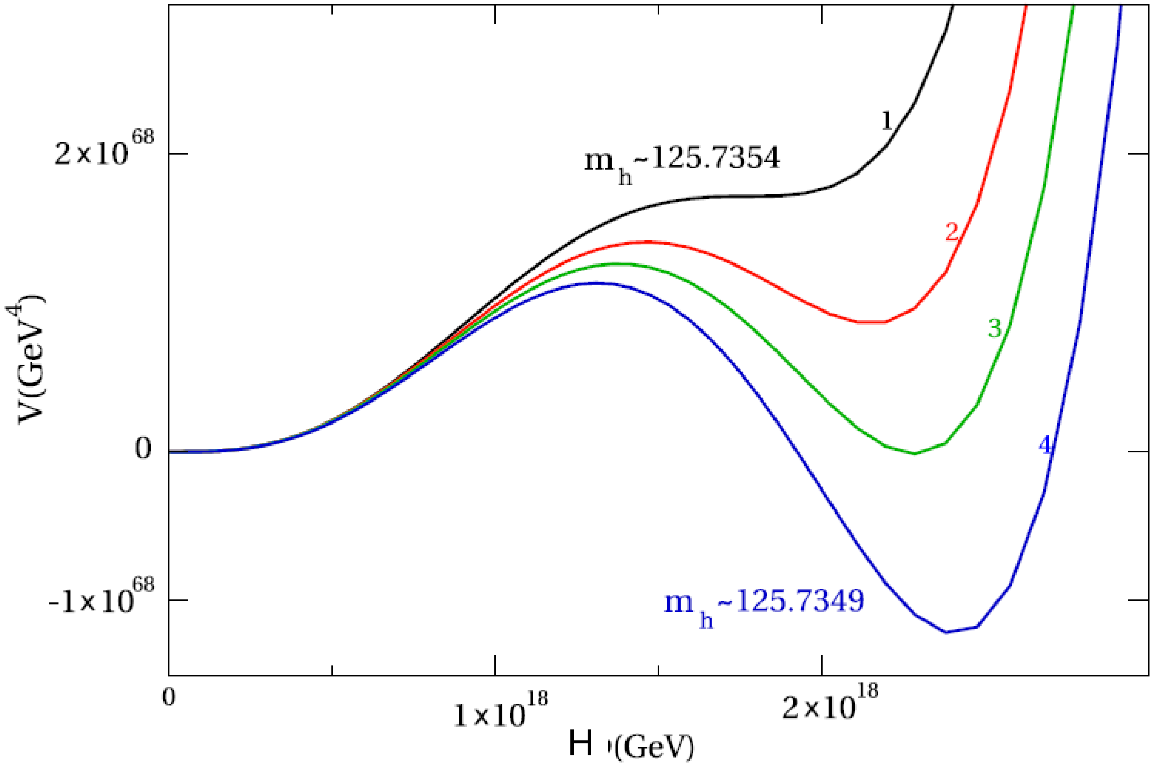
yukawa and majorana parameters subject to RGE

$$\mu_0 = 2\Lambda^2 \frac{f_2}{f_0}$$

aim: flat potential through 2-loop quantum corrections of SM

aim: flat potential through 2-loop quantum corrections of SM

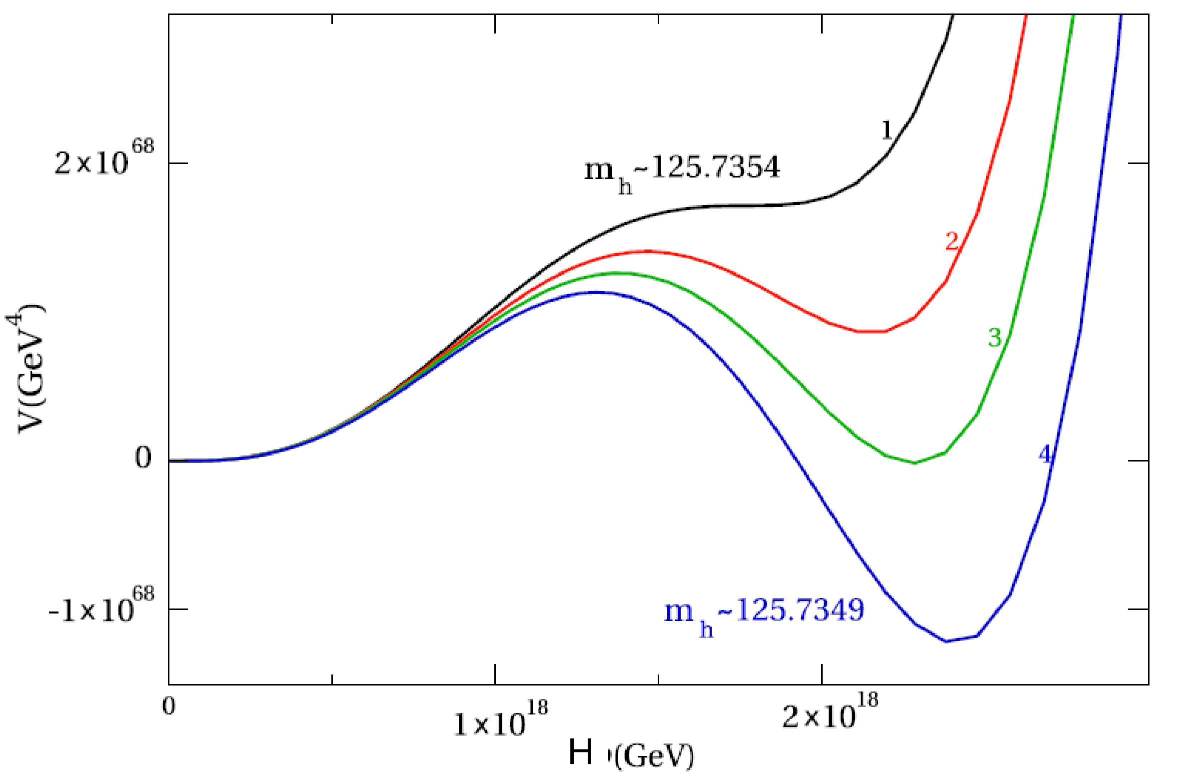
effective potential at high energies: $V(H) = \lambda(H) H^4$



for each value of m_{top} there is a value of m_{higgs} where V_{eff} is on the verge of developing a metastable minimum at large values of $\mathbf{H}$ and V_{higgs} is locally flattened

aim: flat potential through 2-loop quantum corrections of SM

effective potential at high energies: $V(H) = \lambda(H) H^4$



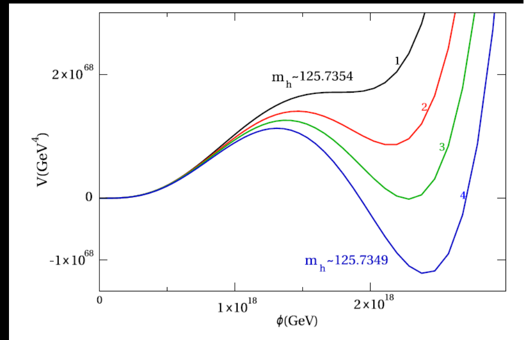
for each value of m_{top} there is a value of m_{higgs} where V_{eff} is on the verge of developing a metastable minimum at large values of **H** and V_{higgs} is locally flattened

approach

- calculate renormalisation of higgs self-coupling
- construct V_{eff} which fits the RG potential around flat region

analytic fit to the higgs potential in the region around the minimum:

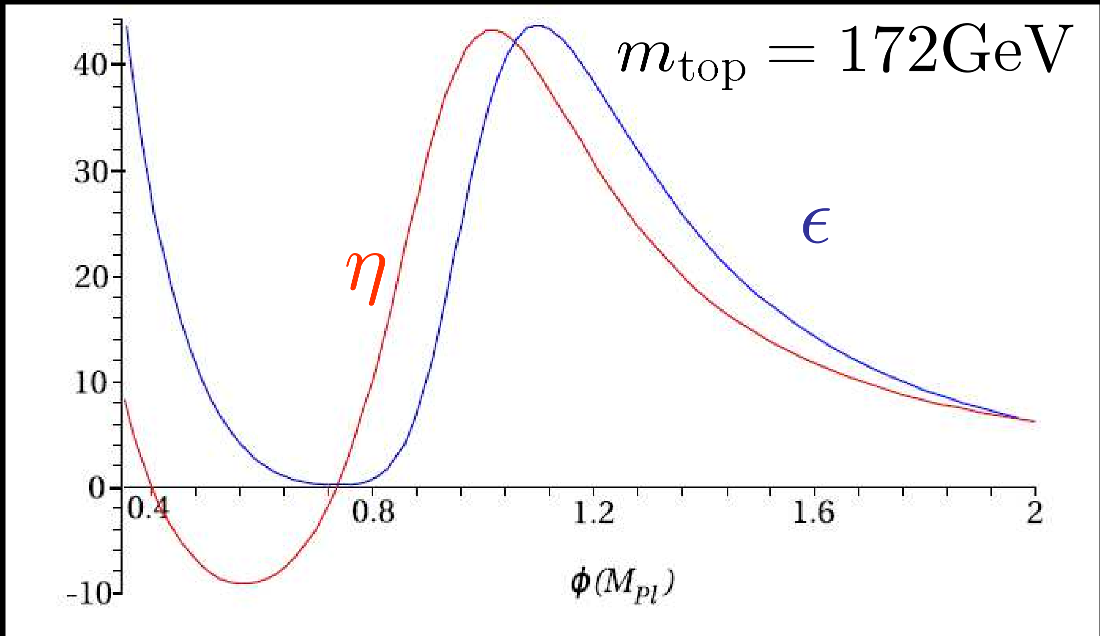
$$\begin{aligned} V^{\text{eff}} &= \lambda_0^{\text{eff}}(H)H^4 \\ &= [a\ln^2(b\kappa_0 H) + c]H^4 \end{aligned}$$



$$\begin{aligned} a(m_{\text{t}}) &= 4.04704 \times 10^{-3} - 4.41909 \times 10^{-5}\left(\frac{m_{\text{t}}}{\text{GeV}}\right) \\ &\quad + 1.24732 \times 10^{-7}\left(\frac{m_{\text{t}}}{\text{GeV}}\right)^2 \\ b(m_{\text{t}}) &= \exp\left[-0.979261\left(\frac{m_{\text{t}}}{\text{GeV}} - 172.051\right)\right] \end{aligned}$$

$c = c(m_{\text{t}}, m_\phi)$ encodes the appearance of an extremum

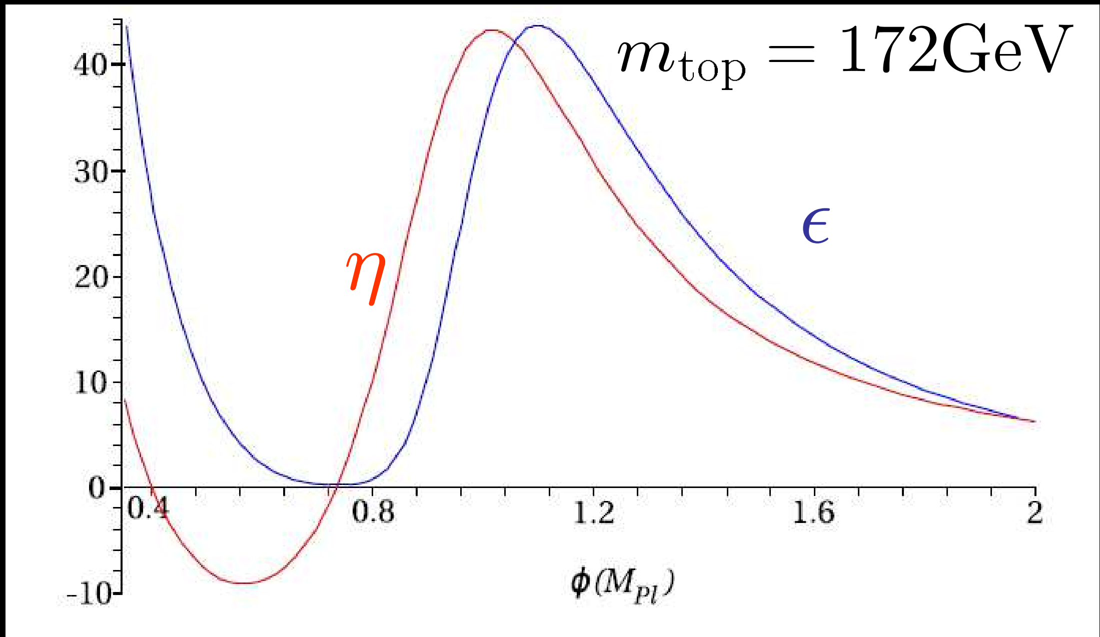
an extremum occurs iff $c/a \leq 1/16$



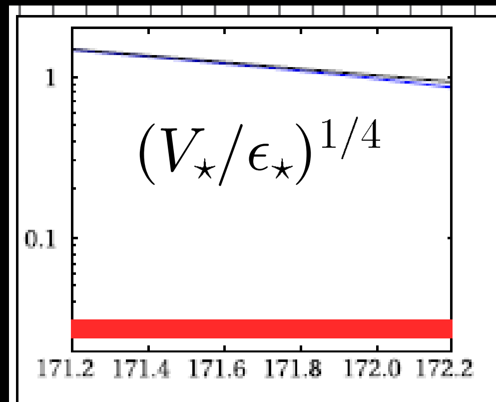
$$N \sim \epsilon^{-1/2}{\rm d}\phi$$

ϵ needs to be too small to allow for sufficient e-folds, and then $(V_\star/\epsilon_\star)^{1/4}$ becomes too large to fit the CMB constraint

buck, fairbairn, sakellariadou, PRD 82 (2010) 043509



$$N \sim \epsilon^{-1/2} \mathrm{d}\phi$$



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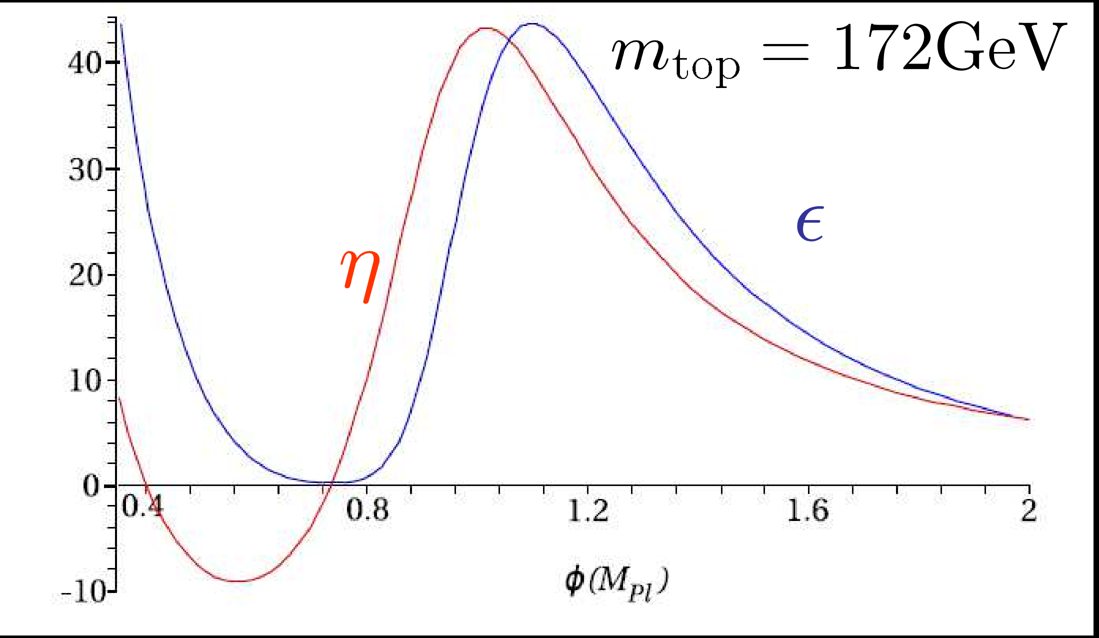
buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

running of the self-coupling at two-loops:

→ slow-roll conditions satisfied BUT

CMB constraints lead to incompatible top quark mass



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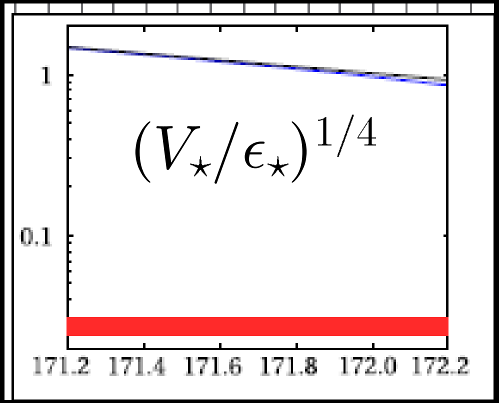

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buck, fairbairn, sakellariadou, PRD 82 (2010) 043509

can we have inflation without introducing a scalar field?

the arbitrary mass scale in the spectral action for the Dirac operator can be made dynamical by introducing a dilaton field,

$$\mathcal{D}/\Lambda \to e^{-\Phi/2}\mathcal{D}e^{-\Phi/2}$$

$$\mathscr{S}_{\rm GDH} = \int \sqrt{G}\left[-\frac{1}{2\kappa_0^2}R + \frac{1}{2}\left(1+\frac{6}{\kappa_0^2 f^2}\right)G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi \ + G^{\mu\nu}D_\mu H'^* D_\nu H' - V_0\left(H'^* H'\right)\right] d^4x$$

f: dilaton decay constant

$$\Phi = (1/f)\tilde{\sigma}$$

dilaton

scalar field

could this dilaton field play the rôle of the inflaton?

chamseddine and connes (2006)

conclusions

how can we construct a quantum theory of gravity coupled to matter ?

- purely gravitational theory without matter

or

- gravity-matter interaction is the most important aspect of dynamics how can we construct a quantum theory of gravity coupled to matter ?

- purely gravitational theory without matter

or

- gravity-matter interaction is the most important aspect of dynamics

below planck scale: continuum fields and an effective action

NCSG:

SM fields and gravity packaged into geometry and matter on a kaluza-klein noncommutative space alain connes' formulation of NCG:

mathematical/physical notions described in terms of spectral properties of operators

aim: differential geometry $\xleftrightarrow{\text{mapping}}$ algebraic terms

topology of space described in terms of algebras

NCSG depends crucially on choice of algebra $\mathcal{A}_{\mathcal{F}}$ represented on a Hilbert space $\mathcal{H}_{\mathcal{F}}$ and the Dirac operator $D_{\mathcal{F}}$

physical picture of the discrete space

- left/right-handed fermions are placed on two different sheets
- higgs fields: the gauge fields in the discrete dimensions
- inverse of separation between the two sheets: EW energy scale

picture similar to the randall-sundrum scenario

4dim brane embedded into 5dim manifold as 3dim brane placed at $x_5 = 0 \;\; , \;\; x_5 = \pi r_{\text{compactification}}$

NCSG extends notion of commutative spaces, using data encoded in a spectral triple on a space composed by $\mathcal{M} \times \mathcal{F}$

- geometric explanation for SM phenomenology

- framework for early universe cosmology