Random Walks in Random environment on trees.

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Motivation :

Does a process in a inhomogeneous but “regular” medium behave roughly the same as in a homogeneous medium.

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- Inhomogeneous $\leftrightarrow$ Random
- Regular $\leftrightarrow$ Space-translation invariant, Ergodic, i.i.d...

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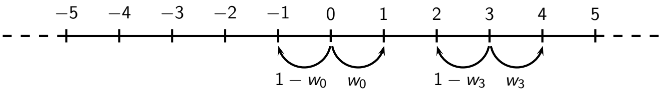

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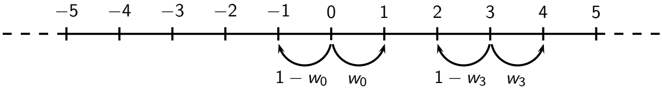

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$$\begin{cases} X_0 = 0 \\ P_w[X_{n+1} = x+1 | X_n = x] = w_x \\ P_w[X_{n+1} = x-1 | X_n = x] = 1 - w_x \end{cases}$$

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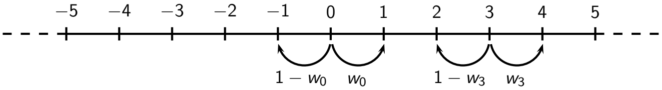

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- $\mu \longrightarrow$ distribution of the environment w,
- $P_w \longrightarrow$ quenched probability,
- $\mathbb{P} = \mu \otimes P_w \longrightarrow$ annealed probability.

Under $\mathbb{P}$, X_n is not a Markov Chain.

Main Results

Theorem (Recurrence/transience; Solomon 1975)

- *If* $E_\mu[log(\frac{1-w_0}{w_0})] < 0$, *then* $\mathbb{P}$-*a.s,* $X_n \to +\infty$,
- *if* $E_\mu[log(\frac{1-w_0}{w_0})] > 0$, *then* $\mathbb{P}$-*a.s,* $X_n \to -\infty$,
- *if* $E_\mu[log(\frac{1-w_0}{w_0})] = 0$, *then* $\mathbb{P}$-*a.s,* $\limsup X_n = +\infty$ *and* $\liminf X_n = -\infty$.

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- The slow regime

Theorem (Sinai 1982)

Suppose $E_\mu[log(\frac{1-w_0}{w_0})] = 0$, $\delta < w_0 < 1-\delta$, $\mu-a.s.$ *for some* $\delta > 0$ *and* $E_\mu[(log(\frac{1-w_0}{w_0}))^2] < \infty$, *then* $\frac{X_n}{(\log(n))^2}$ *converges to some non-degenerate distribution.*

-The "ballistic/diffusive" regime.

Theorem (Solomon 1975; Kesten, Kozlov, Spitzer 1982)

- *If* $E_\mu[\frac{1-w_0}{w_0}] < 1$, *then* $\frac{X_n}{n} \to \frac{1-E[\frac{1-w_0}{w_0}]}{1+E[\frac{1-w_0}{w_0}]}$ $\mathbb{P} - a.s.$
- *If* $E_\mu[\frac{w_0}{1-w_0}] < 1$, *then* $\frac{X_n}{n} \to \frac{1-E[\frac{w_0}{1-w_0}]}{1+E[\frac{w_0}{1-w_0}]}$ $\mathbb{P} - a.s.$
- *If* $1/E_\mu[\frac{1-w_0}{w_0}] \leq 1 \leq E_\mu[\frac{w_0}{1-w_0}]$, *then* $\frac{X_n}{n} \to 0$.

Furthermore, in the last case, if $\kappa > 0$ *is such that* $E_\mu\left[\left(\frac{1-w_0}{w_0}\right)^\kappa\right] = 1$, $E_\mu\left[\left(\frac{1-w_0}{w_0}\right)^\kappa \log^+\frac{1-w_0}{w_0}\right] < \infty$, *and the distribution of* $\log\frac{1-w_0}{w_0}$ *is non-lattice, then*

- *If* $0 < \kappa < 1$ *then* $\frac{X_n}{n^\kappa}$ *converges to an explicit non-degenerate distribution,*
- *If* $\kappa = 1$, *then* $\frac{X_n \log n}{n}$ *converges to an explicit non-degenerate distribution.*

Notations:

Let T be a tree rooted at some vertex e. We call

- $\overleftarrow{x}$ the father of x,
- $|x|$ the distance, or number of edges between x and e,
- moreover we say that $x \sim y$ if x is a neighbor of y
- we call N_x the number of the children $(x_1, x_2, ...x_{N_x})$ of x.

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Let $(w(x,y))_{x,y \in T}$ be a family of random variables such that

$$w(x,y) = 0 \text{ unless } x \sim y$$

$$\forall x \in T, \sum_{y \in T} w(x,y) = 1.$$

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$$w(x,y) = 0 \text{ unless } x \sim y$$
$$\forall x \in T, \sum_{y \in T} w(x,y) = 1.$$

We call Random Walk on (T, w) the Markov Chain defined by

$$\begin{cases} X_0 = e \\ P_T[X_{n+1} = y | X_n = x] = w(x,y). \end{cases}$$

Assumptions

We call $A(x) := \frac{w(\overleftarrow{x}, x)}{w(\overleftarrow{\overleftarrow{x}}, \overleftarrow{x})}$. Note that knowing the $\{w(x,y), y \sim x\}$ is equivalent to knowing the $A(x_i), 1 \leq i \leq N_x$.

Proposition (Neveu, 1986)

given a probability measure q on $\mathbb{N} \otimes \mathbb{R}_+^{*\mathbb{N}^*}$, there exists a probability measure $\mathtt{MT}$ *on the space of marked trees,* $\mathbb{T}$ *such that*

- *the distribution of the random variable $(N_e, A(e_1), A(e_2), \dots)$ is q,*
- *given $\mathcal{G}_n$, the random variables $(N_x, A(x_1), A(x_2), \dots)$, for $x \in T$, $|x| = n$ are independent and their conditional distribution is q.*

Note that the tree T is then a Galton-Watson tree, we will always assume $E_{\mathtt{MT}}[N_x] > 1$. We introduce the (convex, well defined) function

$$\psi(t) = \log E_{\mathtt{MT}}\left[\sum_{i=1}^{N_e} A(e_i)^t\right].$$

and we call as before $\mathbb{P}$ the annealed law.

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Let $p := \inf_{0\leq t\leq 1} \psi(t)$,

Theorem (Lyons/Pemantle 1992[a], F. 2008)

[a] under the additional assumptions that the $A(e_i)$ are i.i.d. and independent of N_e

- *If* $p < 0$ *then the walk is* $\mathtt{MT}$*-a.s. positive recurrent,*
- *if* $p = 0$ *then the walk is* $\mathtt{MT}$*-a.s. recurrent,*
- *if* $p > 0$ *then the walk is* $\mathtt{MT}$*-a.s. transient, conditionally on the event* $\{\, T$ *is infinite* $\}$.

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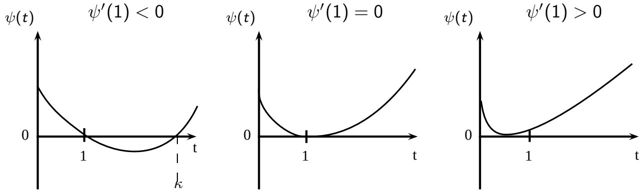
To precise the critical case we must distinguish several cases.



Proposition

Suppose $p = 1$ *and* $\psi'(1) = E_{\mathrm{MT}}\left[\sum_{i=1}^{N(e)} A(e_i)\log(A(e_i))\right]$ *is finite. Then, under some technical assumptions,*

- *if* $\psi'(1) < 0$*, then the walk is a.s. null recurrent, conditionally on the system's survival.*
- *If* $\psi'(1) = 0$ *and for some* $\delta > 0$*,*

  $$E_{\mathrm{MT}}[N(e)^{1+\delta}] < \infty,$$

  then the walk is a.s. null recurrent, conditionally on the system's survival.
- *If* $\psi'(1) > 0$*, and if for some* $\eta > 0$*,* $\omega(x, \overleftarrow{x}) > \eta$ *almost surely, then the walk is almost surely positive recurrent.*

The case $\psi'(1) = 0$

We first study the slow regime, corresponding to the case $\psi'(1) \geq 0$.

Theorem (F., Hu, Shi 2009)

Assume $\psi(1) = \psi'(1) = 0$. *On the set of non-extinction,*

$$\lim_{n\to\infty} \frac{\max_{0\leq k\leq n} |X_k|}{(\log n)^3} = \frac{8}{3\pi^2\sigma^2}, \ \mathbb{P}-a.s.,$$

where

$$\sigma^2 := \mathbb{E}\left\{\sum_{i=1}^{N_e} A(e_i)(\log A(e_i))^2\right\}.$$

The case $\psi'(1) > 0$

Unexpectedly, the case $\psi'(1) > 0$ turns out to be slightly different from the case $\psi'(1) = 0$.

Theorem (F., Hu, Shi 2009)

Assume $\inf_{t\in[0,\,1]} \psi(t) = 0$ *and* $\psi'(1) > 0$. *On the set of non-extinction,*

$$\lim_{n\to\infty} \frac{\max_{0\le k\le n} |X_k|}{(\log n)^3} = \frac{2\,\theta}{3\pi^2\psi''(\theta)}, \quad \mathbb{P} - a.s.,$$

where $\theta \in (0,\,1)$ *is such that* $\psi'(\theta) = 0$ *and* $\psi''(\theta) = \mathbb{E}\Big\{ \sum_{i=1}^{N_e} A(e_i)^\theta \,(\log A(e_i))^2 \Big\}$.

The case $\psi'(1) < 0$

In this case, the behavior depends on $\kappa := \inf\{t > 1;\ \psi(t) > 0\}$.

Theorem (Hu, Shi, 2006[a])

[a] For i.d. $A(x)$, under ellipticity assumptions.

Suppose $p = 0$ and $\psi'(1) < 0$, then

$$\max_{0 \le k \le n} |X_k| = n^{\nu + o(1)},\ \mathbb{P} - a.s.,$$

where

$$\nu = 1 - \frac{1}{2 \wedge \kappa}.$$

The problem of wether $\frac{|X_n|}{n^\nu}$ converges in distribution is still open, however we are able to improve this result when κ is large.

The central limit theorem

We suppose

- $\exists \epsilon_0;\ \epsilon_0 < A(e_i) < \frac{1}{\epsilon_0}\ \forall i,\ a.s.$
- $\forall \alpha \in [0,1],\ E\left[\left(\sum_0^{N(e)} A(e_i)^\alpha\right)\log^+\left(\sum_0^{N(e)} A(e_i)^\alpha\right)\right] < \infty,$
- "$N(e)$ and the $A(e_i)$ a not too dependent"

Theorem (F. 2009)

Suppose $p = 1$, and $\psi'(1) < 0$, then, if $\kappa > 5$, then there is a deterministic constant $\sigma > 0$ such that, under $\mathbb{P}$, the process $\{|X_{\lfloor nt \rfloor}|/\sqrt{\sigma^2 n}\}$ converges in law to the absolute value of a standard brownian motion, as n goes to infinity. Moreover, if $\kappa > 8$, the same holds under P_T, for almost every tree T.

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Definition

- A particle e is initially situated at 0.
- At time 1 it dies and gives birth to a random number N_e of particle e_i, each one having a position $V(e_i)$,
- then each living particle x dies at time 2 and give birth to a random number N_x of particles x_i, with positions $V(x_i)$, in such a way that $(N_x, V(x_i) - V(x))_{1 \leq i \leq N_x}$ has the same distribution as $(N_e, V(e_i))_{1 \leq i \leq N_e}$.

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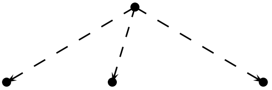

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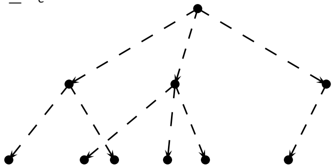

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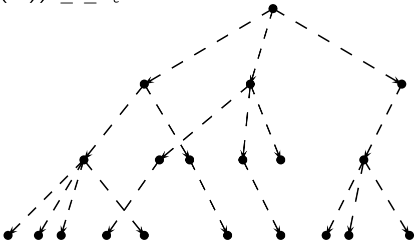

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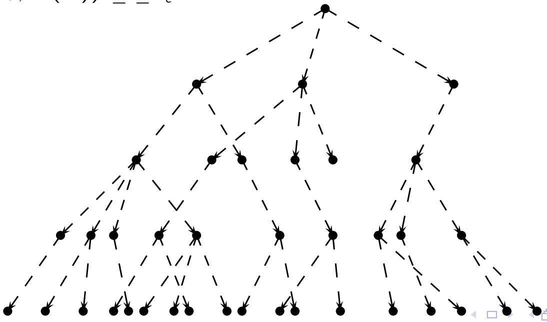

We associate the Branching Random Walk to the marked tree by the relation

$$e^{-V(x)} = \prod_{e<y\leq x} A(y) := C(x).$$

$C(x)$ is called conductance between $\overleftarrow{x}$ and x. We call

$$Y_n^{(\alpha)} := \sum_{x\in T_n} e^{-\alpha V(x)},$$

the Laplace transform of the empirical measure of the BRW. It is closely related to the random walk on T, indeed, denoting by π the invariant measure associated to the walk, we get

$$\pi(x) = \frac{\pi(e)w(e, \overleftarrow{e})}{w(x, \overleftarrow{x})} e^{-\alpha V(x)},$$

where $w(e, \overleftarrow{e})$ is arbitrarily defined as $\frac{1}{\sum_{i=1}^{N_e} A(e_i)}$

Note that $\frac{Y_n^{(\alpha)}}{e^{n\psi(\alpha)}}$ is a positive martingale, therefore it converges almost surely to some variable $Y^{(\alpha)}$. More precisely

Theorem (Biggins, 1977)

Let $\alpha \in \mathbb{R}^+$. Suppose ψ is finite in a small neighborhood of α, and $\psi'(\alpha)$ exists and is finite, then the following are equivalent

- *given non-extinction, $Y^{(\alpha)} > 0$ a.s.,*
- $P_{\mathrm{MT}}[Y^{(\alpha)} = 0] < 1$,
- $E_{\mathrm{MT}}[Y^{(\alpha)}] = 1$,
- $(H1)$: $\forall \alpha \in [0,1]$, $E_q\left[\left(\sum_0^{N(e)} A(e_i)^\alpha\right) \log^+ \left(\sum_0^{N(e)} A(e_i)^\alpha\right)\right] < \infty$, *and* $\alpha\psi'(\alpha) < \psi(\alpha)$ *(H2),*

Consequency:

- if $p < 1$ then $\sum_{x \in T} \pi(x)^\alpha \leq C \sum_{n=1}^{\infty} Y_n^{(\alpha)} < \infty$ as $e^{\psi(\alpha)} < 1$. Therefore the walk is positive recurrent.

Consequency:

- if $p < 1$ then $\sum_{x \in T} \pi(x)^\alpha \leq C \sum_{n=1}^{\infty} Y_n^{(\alpha)} < \infty$ as $e^{\psi(\alpha)} < 1$. Therefore the walk is positive recurrent.
- if $p = 1$, $\psi'(1) \geq 0$, then (H2) is not verified, thus the walk is recurrent
- the other cases are not trivial ...

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We recall the results in the "slow movement" regime

Theorem

Assume $\psi(1) = \psi'(1) = 0$. *On the set of non-extinction,*

$$\lim_{n\to\infty} \frac{\max_{0\le k\le n} |X_k|}{(\log n)^3} = \frac{8}{3\pi^2\sigma^2}, \ \mathbb{P} - a.s.,$$

where

$$\sigma^2 := \mathbb{E}\left\{\sum_{i=1}^{N_e} A(e_i)(\log A(e_i))^2\right\}.$$

Assume now $\inf_{t\in[0,\,1]} \psi(t) = 0$ *and* $\psi'(1) > 0$. *On the set of non-extinction,*

$$\lim_{n\to\infty} \frac{\max_{0\le k\le n} |X_k|}{(\log n)^3} = \frac{2\,\theta}{3\pi^2\psi''(\theta)}, \ \mathbb{P} - a.s.,$$

where $\theta \in (0,\,1)$ *is such that* $\psi'(\theta) = 0$

We call τ_n the hitting time of T_n and τ_e the first return to the root. Let $\rho_n := P_T(\tau_n < \tau_e)$

Theorem

Assume $\inf_{t\in[0,1]} \psi(t) = 0$ *and* $\psi'(1) \geq 0$. *Almost surely on the set of non extinction, (i) if* $\rho_n \geq e^{-(c_1+o(1))n^{1/3}}$ *for some positive constant* c_1*, then*

$$\liminf_{n\to\infty} \frac{1}{\log^3 n} \max_{0\leq k\leq n} |X_k| \geq c_1^{-3};$$

(ii) if $\rho_n \leq e^{-(c_2+o(1))n^{1/3}}$ *for some positive constant* c_2*, then*

$$\limsup_{n\to\infty} \frac{1}{\log^3 n} \max_{0\leq k\leq n} |X_k| \leq c_2^{-3};$$

Therefore the proof reduces to showing that $\rho_n = e^{-(a^*+o(1))n^{1/3}}$ for some appropriated a^*.

A lower bound in the case $\psi'(1) > 0$

Note that $P_T(\tau_n < \tau_e) \geq \max_{|x|=n} P_T(\tau_x < \tau_e)$. The last probability can be computed explicitly (it is a 1-dimensional Random Walk) , and is equal to

$$\frac{w(e, x_1)e^{V(x_1)}}{\sum_{e<z\leq x} e^{V(z)}}$$

therefore

$$\rho_n \geq \frac{c(T)}{n} e^{-\min_{|x|=n} \max_{e<z\leq x} V(z)} := \frac{c(T)}{n} e^{-\min_{|x|=n} \overline{V}(z)}$$

This can be estimated.

Theorem

Assume $p = 1$ *and* $\psi'(1) \geq 0$. *Let* $\theta \in (0, 1]$ *be such that* $\psi'(\theta) = 0$. *We have, on the set of non-extinction,*

$$\lim_{n\to\infty} \frac{1}{n^{1/3}} \min_{|x|=n} \overline{V}(x) = \left(\frac{3\pi^2\sigma_\theta^2}{2}\right)^{1/3}, \qquad \mathbb{P}\text{-a.s.},$$

where

$$\sigma_\theta^2 := \frac{1}{\theta}\mathbf{E}\Big\{ \sum_{|x|=1} V(x)^2 e^{-\theta V(x)} \Big\}.$$

This gives a first bound for the walk. Unfortunately, it is optimal is the case $\rho'(1) > 0$ but not in the case $\rho'(1) = 0$.

A lower bound in the case $\psi'(1) = 0$

For any x in T we call $T^{(k)}(x)$ the k-th visit at x. Since the walk is recurrent, each $T^{(k)}(x)$ is well-defined; we have

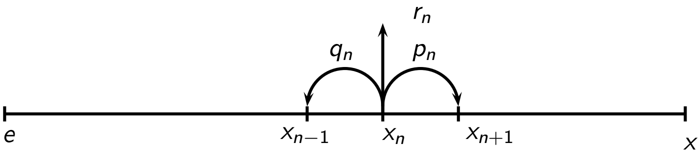
$$\begin{aligned} \varrho_n &= \sum_{|x|=n} \sum_{k=1}^{\infty} P_\omega \Big\{ T^{(k)}(x) < \tau_e < T^{(k+1)}(x), \max_{T^{(k)}(x) < i \le \tau_e} |X_i| < n \Big\} \\ &= \sum_{|x|=n} \sum_{k=1}^{\infty} P_\omega \Big\{ T^{(k)}(x) < \tau_e, \max_{T^{(k)}(x) < i \le \tau_e} |X_i| < n \Big\}. \end{aligned}$$

Applying the strong Markov property at $T^{(k)}(x)$, we see that the probability on the right-hand side equals $P_\omega\{T^{(k)}(x) < \tau_e\} \, P_\omega^x\{\tau_n > \tau_e\}$. Therefore,

$$\varrho_n = \sum_{|x|=n} P_\omega^x\{\tau_n > \tau_e\} \sum_{k=1}^{\infty} P_\omega\{T^{(k)}(x) < \tau_e\} = \sum_{|x|=n} P_\omega^x\{\tau_n > \tau_e\} \pi(x)$$

By a spinal decomposition, we can reduce to studying $P_\omega^x\{\tau_n > \tau_e\}$ for a slightly different law of environment, with a spine. We reduce to studying a walk on $[e, x]$ defined as follows: when the walk is at some point $x_n \in [e, x]$,

- it goes to x_{n+1} with probability $p_n = w(x_n, x_{n+1})$
- it goes to x_{n-1} with probability $q_n = w(x_n, x_{n-1})$
- it dies with probability $r_n = P_T^x(\tau_n < \tau_x, X_1 \notin \{x_{n-1}, x_{n+1}\})$



We obtain, after some computations, that when $\psi(1) = \psi'(1) = 0$,

$$\liminf_{n\to\infty} \frac{\max_{0\le k\le n} |X_k|}{(\log n)^3} \ge \frac{8}{3\pi^2\sigma^2}, \qquad \mathbb{P}\text{-a.s..} \tag{1}$$

However this method relies on $\sum_{|x|=n} C(x)$ being a martingale, which is not the case when $\psi'(1) > 0$. The other bounds are obtained by taking well-chosen cutsets.

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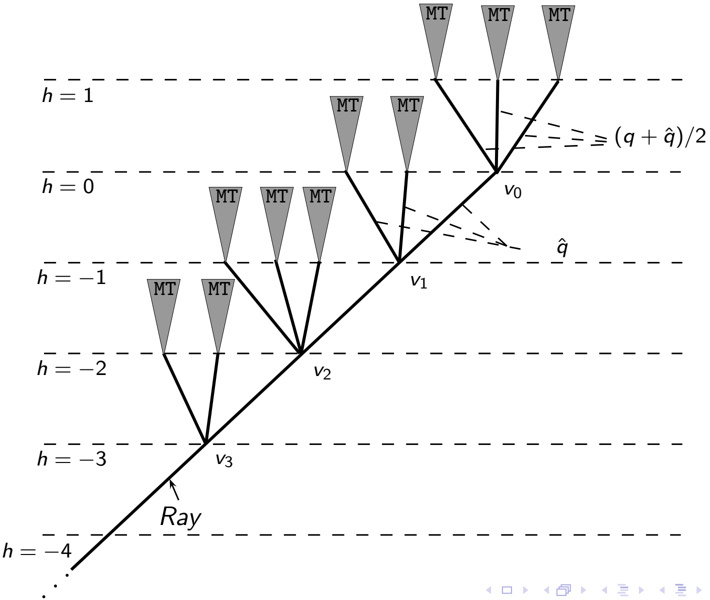
We study here the case $p = 1$, $\psi'(1) < 0$. Recall that in this case, $E[\sum_{i=1}^{N_x} A(e_i)] = 1$. We first introduce a new distribution on trees. We call MT the usual distribution of Marked Trees. We call q the distribution of $(N_e, A(e_i))$ and $\hat{q}$ the distribution defined a by $\frac{d\hat{q}}{dq} = \sum_{i=1}^{N_e} A(e_i)$. We construct a marked graph the following way.

- We set an infinite ray $Ray\ e = v_0, v_1, \ldots$. such that $v_{i+1} = \overleftarrow{v_i}$.
- To each v_i, $i \neq 0$, we attach a set of children with distribution $\hat{q}$.
- To e we attach we attach a set of children with distribution $(\hat{q} + q)/2$.
- Finally to all the vertices not on Ray we attach independent trees with law MT

We also introduce the horocycle distance h, defined as $h(e) = 0$ and $h(\overleftarrow{x}) = h(x) - 1$.


MT
$h = 1$
$= (q + \hat{q})/2$
$h = 0$
v_0
$\hat{q}$
$h = -1$
v_1
$h = -2$
v_2
$h = -3$
v_3
Ray
$h = -4$

We can define as before a Random Walk on a $\mathtt{IMT}$ tree. We call T_t the tree "shifted" at X_t.

Proposition

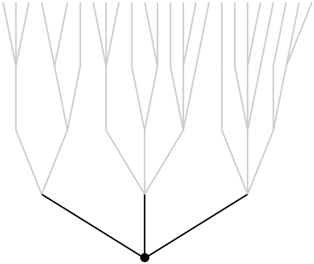
The process T_t is invariant.

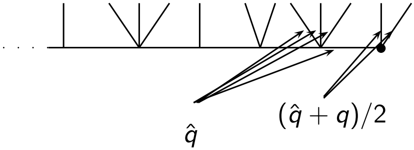
For this measure, things behave quite well

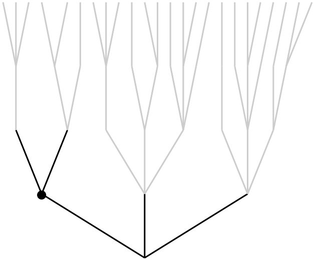
Theorem

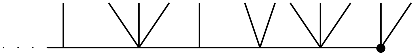
Suppose $p = 1$, $\rho'(1) < 0$ and $\kappa \in [2, \infty]$. There exists a deterministic constant σ such that, under $\mathbb{P}_{\mathtt{IMT}}$ the process $\{h(X_{\lfloor nt \rfloor})/\sqrt{\sigma^2 n}\}$ converges in distribution to a standard brownian motion, as n goes to infinity. Moreover, if $\kappa > 5$, then we have a quenched CLT.

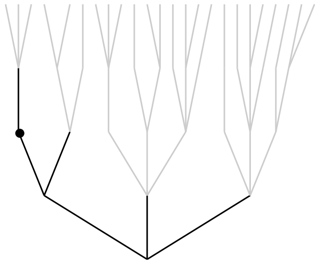
It remains to go back to the original process, by a coupling method.

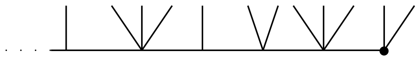


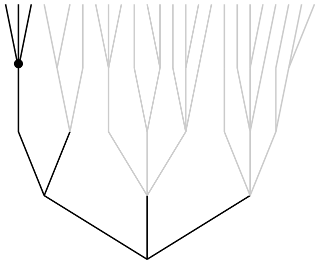

$\hat{q}$
$(\hat{q}+q)/2$

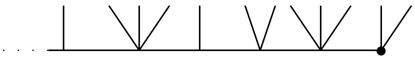


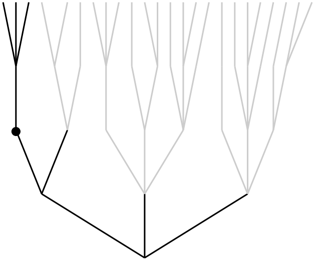


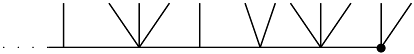


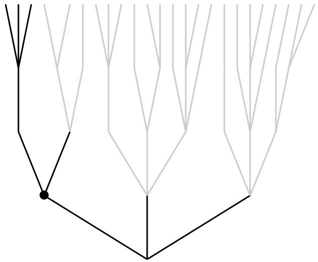


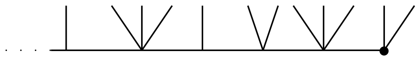


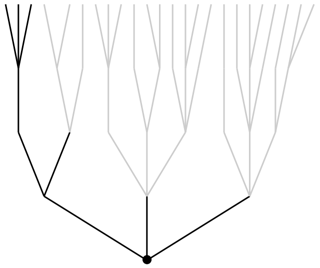


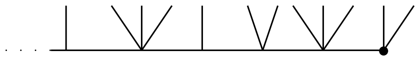


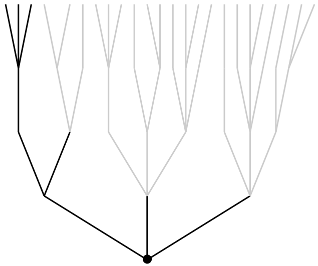


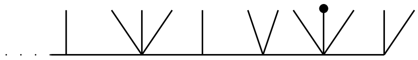


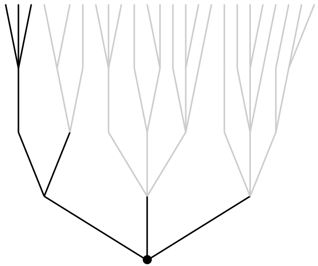


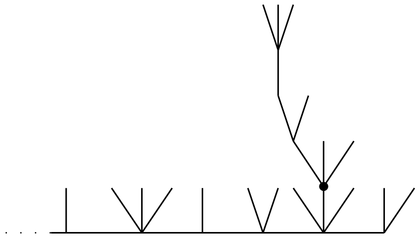


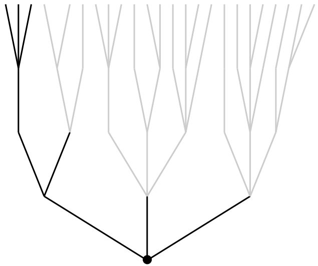


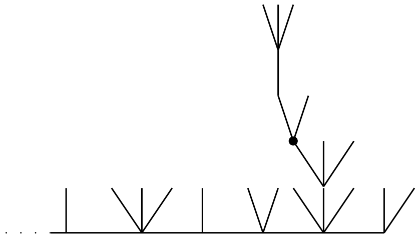


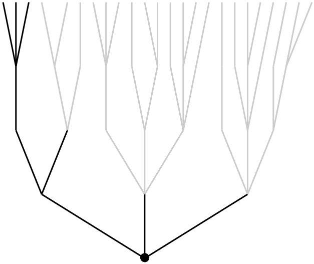


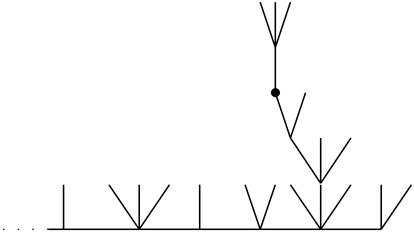


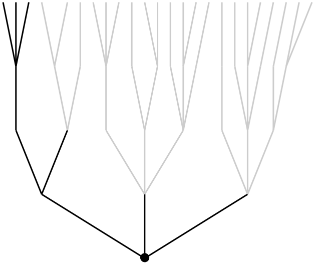


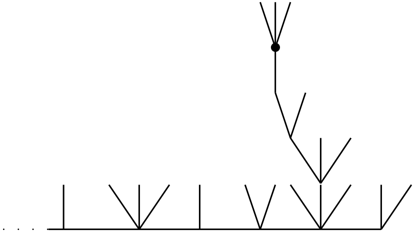


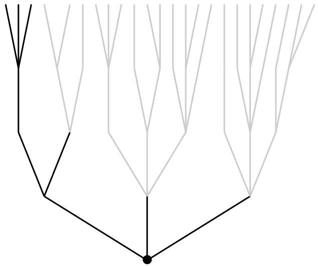


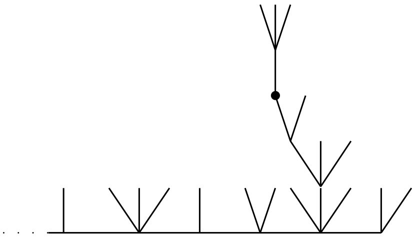


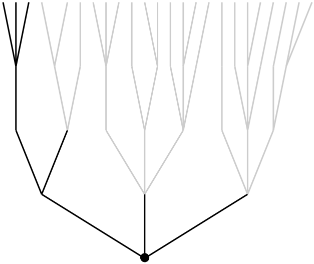


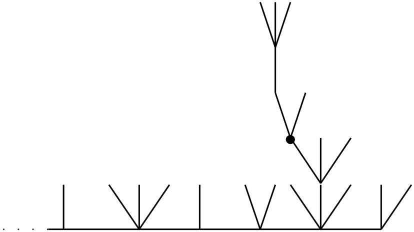


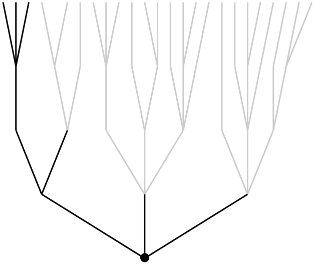


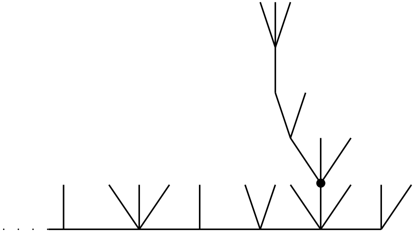


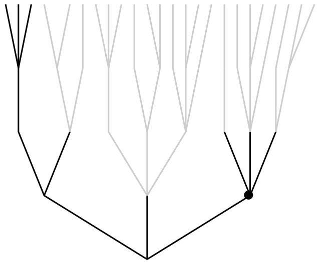


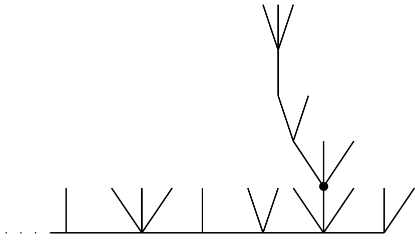


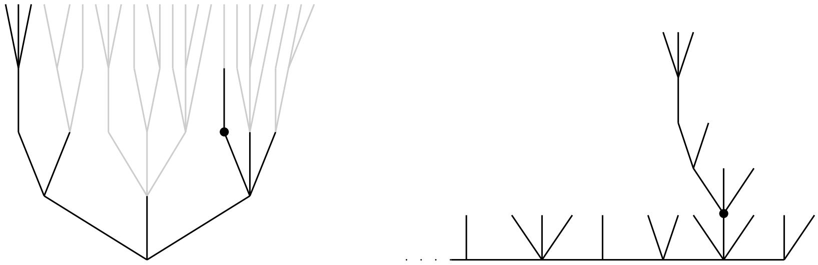


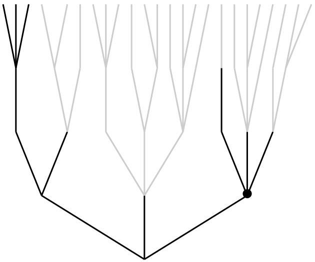


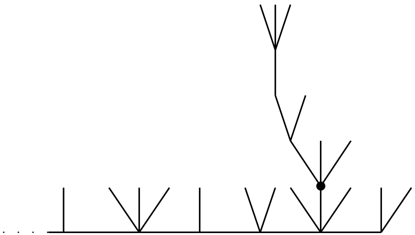


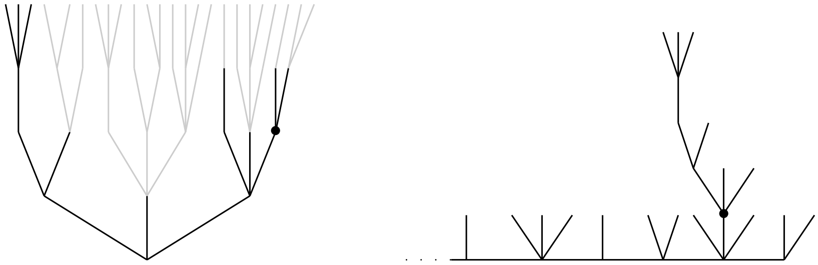


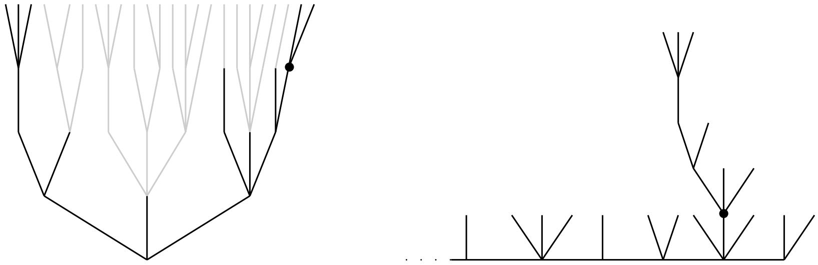


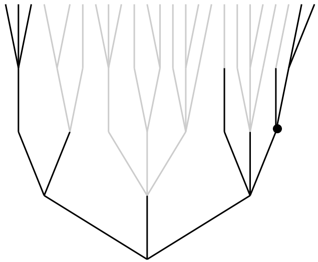


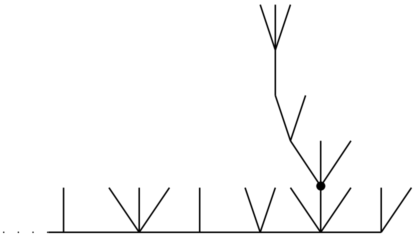


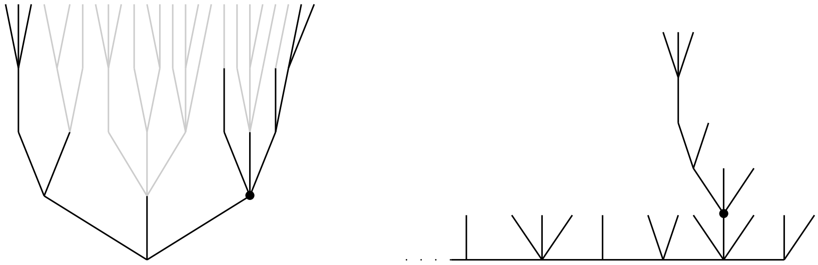


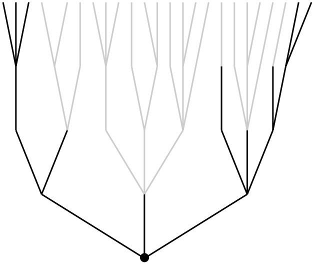


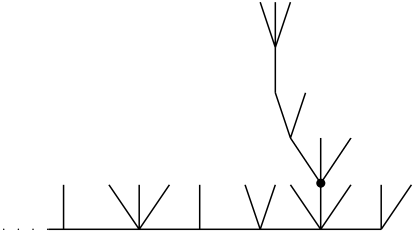


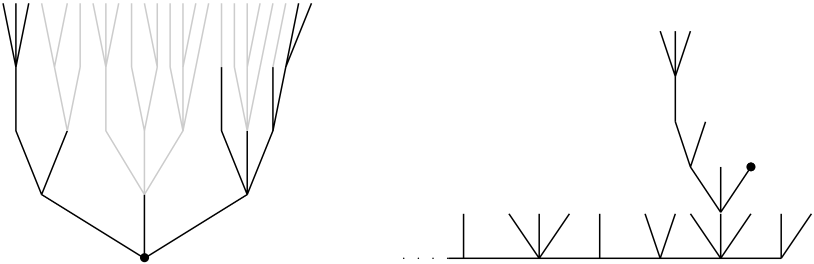


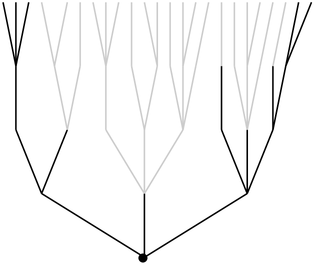


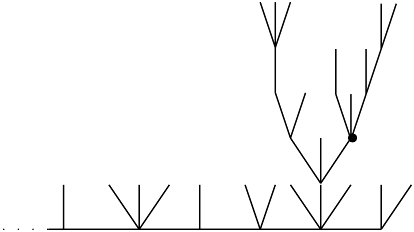


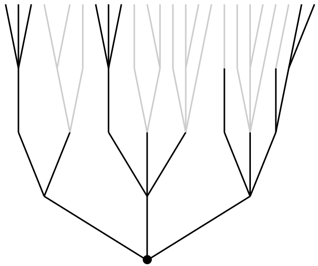


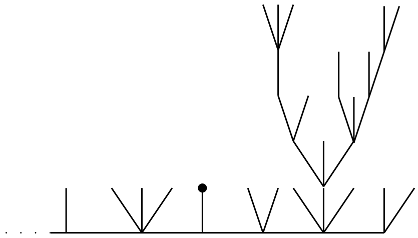


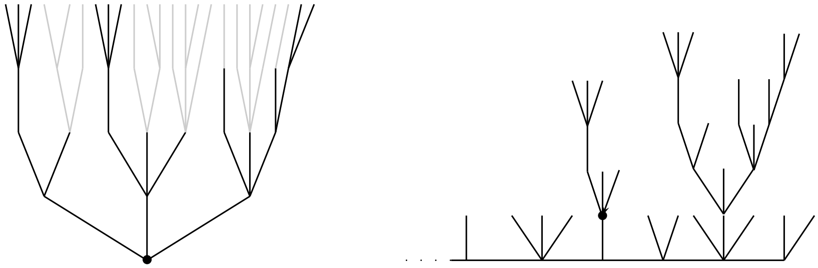












- We do obtain a Random path on a IMT tree, whose excursions are coupled with those of the original walk.
- However, a certain error comes from the parts outside those excursion.
- The method to bound this error is quite delicate, and we loose here some precision on the results.

Conclusion : Open questions

- Does $\frac{|X_n|}{n^\nu}$ converge in law for all κ, when $\psi'(1) < 0$
- How does $|X_n|$ behave in the slow regime, and a related issue would be, where is $\min_{|x|=n} \overline{V}(z)$ realized.

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