Postdoctoral Position

Control of dynamical systems with uncertainty
Theoretical and numerical issues

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Location: Laboratoire de Mathématiques Blaise Pascal (http://recherche.math.univ-bpclermont.fr/)
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Salary: The net salary is 2021.88 euros.
Starting date: between September 1st, 2017 and January 1st, 2018.
Duration: 12 months with possible renewal for 12 additional months.
Required qualification: Applicants should have a PhD in Applied Mathematics (automatic control theory, dynamical systems, partial differential equations), Probability or Scientific computing.
General topics: Partial differential equations, Control theory, Numerical approximation, Probability.
Mobility: The position includes travel facilities in Europe in order to collaborate with partners team, in particular the MC3 group in Spain.

Scientific context. The topic of controllability of distributed parameter systems has attracted the interest of many researchers and important progress has been made during the last decades. The context of distributed systems controllability can be briefly summarized as follows: let X a Banach space, t the time variable and $\mathbf{x} = \mathbf{x}(t) : [0, T] \to X$ solution of the following dynamical system :

$$\begin{cases} \dot{\mathbf{x}}(t;\xi) = A_\xi \mathbf{x}(t;\xi) + B_\xi \mathbf{u}(t) + f_\xi(t), \quad t > 0, \\ \mathbf{x}(0;\xi) = \mathbf{x}_{0,\xi}, \end{cases} \tag{1}$$

starting at the point $\mathbf{x}_{0,\xi}$. The main issue in control theory is to act on the system through an additional control function $\mathbf{u}(t)$ such that the state $\mathbf{x}(t)$ reaches a given target $\mathbf{x_T}$ at a final controllability time T, that is $\|\mathbf{x}(T,\xi) - \mathbf{x_T}\|_X = 0$. This is illustrated in Figure 1. This theory where we look for deterministic

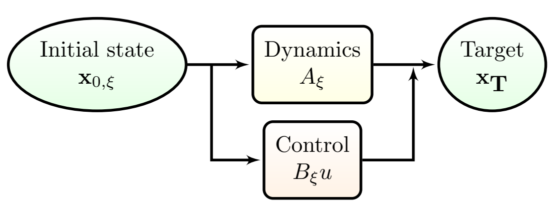

Figure 1: Schematic view of a controlled system.

controls is now very well-established, both at the theoretical level and at the practical level of the numerical approximation (see [1],[2],[3],[4]). Most of the research assumes that the input data of the system (here the external force f_ξ, the initial condition $\mathbf{x}_{0,\xi}$ as well as coefficients in the operators A_ξ and B_ξ) under consideration are perfectly known. However, this assumption is unrealistic with respect to real-world applications. Indeed, in practice, input data of physical systems such as material properties, applied loads, boundary conditions and geometry are affected by a modicum of uncertainty, which should be accounted for in more realistic mathematical models. Exact controllability of dynamical systems, in particular partial differential equations (PDEs) with random input data (in particular, parameter-dependent PDEs) is an emergent topic. Recently, the concept of averaged control has been introduced in [5] in a finite dimensional setting, where the quantity of interest to be exactly controlled is the average of the system response with respect to the random parameter. For system (1) where input data may depend on the parameter variable ξ (ξ can be for instance a random variable), the average controllability reads as follows :

$$\mathbb{E}(\mathbf{x}(T,\xi)-\mathbf{x_T}) := \int_\Omega \|\mathbf{x}(T,\xi)-\mathbf{x_T}\|_X dP_\xi = 0,$$

($\mathbb{E}(z_\xi)$ denotes the mean of the random variable z_ξ with respect to a probability measure $(P_\xi)_{\xi\in\Omega}$).

At the theoretical level, some positive averaged controllability results have been obtained for finite dimensional system [6] as well as for infinite dimensional system [7]. This later case, which concerns notably PDEs is still in its infancy since results concerning the existence of average controls are known only in very particular situations assuming that the operator A_ξ only depend on the random variable. The more relevant case for applications where the operator A_ξ depends both on the spatial variable and on the random variable is still open. Another very important issue, which must be addressed in control of random PDEs, is the one of robustness. It means that, in addition to controlling in average the state variable of the system, one should ensure that the control is less sensitive to fluctuations of the PDE random input data. Inspired in probability theory, higher order moments, for instance the variance of the variable to be controlled, may be used as a measure of robustness. This is an open problem in the context of exact controllability for random PDEs.

At the numerical approximation level, null controls are usually obtained introducing objective functions and gradient methods. For example, we mention [8] where in the deterministic case, a convergent approximation method is described and implemented to obtain null controls for the wave equation. For random PDEs (even for which null averaged controllability result holds true), there are not equivalent results in the literature. We mention however the recent contributions [9,10] in the closed context of optimal control for the heat and the beam equation respectively.

Research project. The main objective is to study the average controllability of simple linear partial differential equations, both on a theoretical and numerical viewpoint. A typical but still open example is the following one:

$$(2)\qquad \begin{cases} \ddot{\boldsymbol{w}}(t,x;\xi) - a(x;\xi)\Delta^2\boldsymbol{w}(t,x;\xi) = \chi_{\mathcal{O}}(x;\xi)\boldsymbol{u}(x,t) & (t,x)\in(0,\infty)\times\Omega \\ \boldsymbol{w}(t,x;\xi) = 0, & (t,x)\in(0,\infty)\times\partial\Omega \\ \Delta\boldsymbol{w}(t,x;\xi) = 0, & t\in(0,\infty)\times\partial\Omega \\ \boldsymbol{w}(0,x;\xi) = w_0(x), \quad \dot{\boldsymbol{w}}(0,x;\xi) = w_1(x), & x\in\Omega, \end{cases}$$

which models the vibrations of simply supported elastic plate submitted to initial condition (w_0, w_1) [3]. The function $a = a(x;\xi)$ takes into account the mechanical property of the plate and is submitted to uncertainty. For any non empty subset $\mathcal{O}$ of the open set Ω, we look for a control function $\boldsymbol{u}$ acting on $\mathcal{O}$

such that

$$\mathbb{E}\Big(\boldsymbol{w}(T,\cdot;\xi), \dot{\boldsymbol{w}}(T,\cdot;\xi)\Big) = 0$$

for some time $T > 0$. For this kind of problem, the following issues may be addressed:

- Proof of existence of average controls;
- Set up and study of a method/algorithm of approximation of average control;
- Numerical analysis of the method and implementation;
- Numerical study of the influence of the uncertainty on the average control.

The postdoctoral stay could be scheduled as follows:

- Bibliographical study of control theory for the deterministic case [1].
- Bibliographical study of controllability result for parameter dependent PDEs [5].
- Bibliographical study to approximate numerically stochastic PDEs [11,12,13] and then to approximate null controls for the deterministic case [8].
- Study of the average controllability for parameter dependent linear PDEs and proof of existence of null averaged controls.
- Numerical approximation of such average null controls, implementation and analysis.

This program will be adapted according to the experience of the candidate.

Interested person may contact and send by email a CV to Arnaud Munch and/or Nourddine Azzaoui and/or Nicolae Cindea. The position will remain open until filled.

Bibliography

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